Rationing Can Backfire: The “Day without a Car” in Mexico City

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A ban restricting each car from driving on a specified weekday is found to have increased total driving in Mexico City. Because of the ban, cars effectively represent “driving permits,” and some households have bought an additional car and increased their driving. Greater use of old cars, congestion effects, and increased weekend driving may also have contributed to the disappointing results. The ban has high welfare costs and does not deliver the intended benefits of reduced driving—quite the contrary.

The experience provides an interesting lesson in applied welfare economics. Theory indicates that this is a costly way of reducing traffic and pollution. But the finding that the strategy is counterproductive could be made only with applied quantitative analysis.

In November 1989 the Mexico City administration imposed a regulation banning each car from driving a specific day of the week. Called Hoy no circula (this one does not circulate today), the “Day without a Car” regulation specifies that cars with license plate numbers ending with digits 0 or 1 do not drive on Monday, 2 or 3 do not drive on Tuesday, and so on. Restrictions do not apply on weekends. The regulation applies to all cars (except those of the fire department) and thus to firms as well as households. We use the term household for simplicity. Compliance is generally believed to be high: the police are visible and fines are heavy.

The regulation has been both popular and controversial. Some people argue that it places a reasonable burden on car owners in order to alleviate congestion and pollution problems. Others argue that it is inefficient and unfair: inefficient in the way most rationing devices are inefficient; unfair because it is more easily avoided or accommodated by some than by others. Finally, some people are arguing that the regulation is counterproductive, actually increasing the levels of congestion and pollution because many households have purchased additional cars to circumvent the ban.

This article analyzes the policy pragmatically. Section I shows how the results of rationing can be compared with those that would be obtained using market-
based instruments. We illustrate why rationing entails welfare costs at least as high as those of a market-based mechanism producing the same reduction in trips.

Section II presents an empirical framework for estimating the reductions in demand induced by the regulation. A model of gasoline demand is estimated using aggregate time-series data from before the regulation. This model is used to simulate a counterfactual scenario for demand in subsequent periods as if the regulation had not been introduced.

The results of these simulations motivate further examination of three aspects of car ownership and trip generation. First, due to the integer nature of cars and the fact that cars effectively come bundled with "workday driving permits," households may want an additional car once theirs is made less useful by the regulation. Second, multiple drivers in a household could mean that total car use increases even though an additional car is purchased primarily to substitute for the household's existing car on its banned day. Third, increased congestion, substitution between trips, and differences in fuel efficiency all affect gasoline consumption per car. These issues are pursued in the following sections.

Section III presents a model of car ownership based on household survey data. The regulation is modeled as a reduction in the service flow from each car. Some car-owning households would want an additional car to compensate for the lost service flow from the car they already own, while others—with lower incomes—would now find their car insufficiently useful and want to sell it. Our model indicates that the two groups of households should be of about the same size, with a few more car sellers than buyers. Thus, our model of car ownership is unable to capture the apparent reality of increased ownership. We proceed to discuss some known weaknesses of the car ownership model. If there are some transaction costs in the market for used cars, then our model will overestimate the number of "sellers" in Mexico City.

Section IV considers additional information indicating major changes in the market for used cars. Mexico City traditionally exported used cars to the rest of the country but started importing them in the years following passage of the regulation. The role of used cars in the response of Mexico City households is significant in itself because increased use of old cars implies that gasoline consumption has increased more than total driving and that pollution has increased more than has gasoline consumption. We also examine the possibility that changes in driving patterns—increased driving on weekends—may have contributed to increased driving per car. This is relevant because some advocates of the regulation maintain that beneficial changes in driving patterns could justify the regulation even if reductions in total driving never materialize.

I. Market-Based and Regulatory Demand Management

It is well known that pollution charges are first-best instruments because they achieve reductions with the lowest possible losses in welfare. However, often—and sometimes with good reason, as when the costs of monitoring individual
emissions are high—such instruments are not used. Eskeland (1994) and Eskeland and Devarajan (1996) discuss how many real-world automobile pollution control strategies could be improved by including instruments that directly discourage car use, such as fuel taxes. Improvement is possible because existing programs provide incentives to make cars and fuels cleaner (standards) but fail to discourage the use of cars. Fuel taxes could be effective in Mexico because demand elasticities for gasoline are estimated to be in the range of -0.8 to -1.25 (see Berndt and Botero 1985 and Eskeland and Feyzioglu 1997). Instruments to reduce pollution by reducing polluting trips include gasoline taxes, driving bans, parking fees, highway tolls, and subsidies for public transport. But what are the welfare costs for consumers who sacrifice trips in response to demand management instruments? We make the simplifying assumption that income can be transferred costlessly between households and between the private and the public sectors, allowing us to abstract from income distribution effects and any premium (or penalty) on public revenue generation.

When a trip is sacrificed due to an increase in the tax on gasoline, the value of the sacrificed unit to the consumer is the retail price of gasoline. Thus although some inframarginal units of gasoline (and trips) are worth more to consumers, a higher gasoline tax systematically screens out the trips that are worth the least. This property allows the gasoline tax to reduce trips at the lowest possible welfare cost.

Demand reductions resulting from a regulation rarely are this selective. The “Day without a Car” program may curtail trips in households with a very high willingness to pay, and it may block a household’s driving on Tuesday, say, even if the household could more easily have sacrificed other trips. These effects result from the fact that the regulation does not allow trading of the rationed commodity, thus curtailing both inframarginal and marginal trips. (Goddard forthcoming advocates solving part of this problem by making driving permits tradable.) Figure 1 compares the welfare losses from a regulation and a tax increase, with the two calibrated to give the same reductions in demand.

With the rationing mechanism used in Mexico City, issues are slightly more complex because the effects on demand are not known. First, the ration applies to the utilization of a vehicle that was not fully utilized (24 hours a day, 7 days a week) at the outset. For this reason, if users can move trips from one day to another or exchange car services with others—on Tuesdays I drive twice my distance to pick you up, and on Thursdays you return my favor—vehicle kilometers and the number of vehicles may both remain constant. Second, households can purchase an additional car, thereby purchasing four “workday driving permits” and two “weekend permits.” All of these escape routes place upper bounds on the costs of compliance for a household. Finally, redistribution of trips between weekdays, when restrictions apply, and weekends, when no restrictions apply, affects use. Increased weekend driving would limit the curtailment of total driving but might be valued for redistributing traffic to less congested days.
II. Aggregate Gasoline Consumption

In this section, we investigate the behavior of aggregate gasoline consumption in the Mexico City Metropolitan Area (MCMA) before and under the ban. We trace the consumption pattern with quarterly observations from January 1984 through December 1992. Actual consumption levels are given by the solid line in figure 2. The driving ban became effective at the end of 1989, where the dotted lines appear. We assume that aggregate gasoline consumption in Mexico City depends on the gasoline price and household income. Other variables like congestion, quality of cars, and number of cars are not available.\(^1\)

The equation for gasoline consumption, \(c_t\), is:

\[
(1) \quad c_t = \alpha_0 + \alpha_1 p_t + \alpha_2 y_t + e_t \quad t = 1, \ldots, T
\]

where \(p_t\) is the weighted average of gasoline prices in constant pesos (types of gasoline, by share in total use), and \(y_t\) is a proxy for personal income in Mexico City. Gasoline consumption is total sales of all types from terminals in the metropolitan area, in millions of liters (diesel is used only for heavy-duty vehicles in Mexico). Outgoing international telephone calls from Mexico City are used as a proxy for income. All variables are in logarithms. Thus \(\alpha_1\) is a price elasticity, and \(\alpha_2\) is a transformed income elasticity (transformed because the proxy for income may itself have an income elasticity different from 1).

The hypothesis of the policymakers and the analysts is, of course, that imposition of the restriction changes consumption patterns, that is, shifts the demand function (equation 1). Such a change can be a change in the constant term, \(\alpha_0\), or a change in the elasticities, \(\alpha_1\) and \(\alpha_2\), without any change in \(\alpha_0\), or both. To

\(^1\) Eskeland and Feyzioglu (1997) discuss demand estimation based on annual data and a national focus.
capture these possible changes, it is standard to introduce a dummy variable that is 0 before the restriction is imposed and 1 after. The estimated coefficients for this dummy variable and its interaction with price and income indicate whether any statistically significant changes in the demand function are related to the restriction.

First we analyze the time-series properties in terms of nonstationarity for the data on gasoline consumption, gasoline price, household income, and the residual (see appendix A). We conclude that, for our purposes, it is sufficient to model the variables as stationary, and that ordinary least squares estimation is appropriate. Second, to test the hypothesis that the demand function has not changed, we estimate equation 1 with a dummy variable for the periods under regulation. The estimated elasticities are given in table 1.

The key result is that a significant change in the gasoline demand function is associated with the regulation. The effect of the regulation on the demand function for the relevant income and price ranges can be seen in figure 3. The constant term shifts downward, but in the relevant area the demand function under regulation is above the demand function without regulation. This is due to an increase in the absolute values of the price and income elasticities. For sensitivity analysis, when a model is estimated allowing only the constant term to change, this term increases significantly, confirming that an increase in gasoline consumption is the result of the regulation.
Table 1. Elasticities from the Estimated Gasoline Demand Function, Mexico City, 1983-92

<table>
<thead>
<tr>
<th>Variable</th>
<th>Without regulation</th>
<th>Under regulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>7.27**</td>
<td>5.30†</td>
</tr>
<tr>
<td>Gasoline price</td>
<td>-0.17*</td>
<td>-0.05</td>
</tr>
<tr>
<td>Income</td>
<td>0.06**</td>
<td>0.24†</td>
</tr>
</tbody>
</table>

* Significantly different from zero at 5 percent.
** Significantly different from zero at 1 percent.
† Significantly different from the "without regulation" model at 1 percent.

Source: Authors' calculations.

To estimate how much gasoline consumption has increased because of the ban we simulate a counterfactual scenario for the periods of regulation. We use the demand function based only on data from periods prior to regulation and insert actual values for price and income for 1990 to the end of 1992 to simulate what gasoline consumption would have been without a ban. In other words, we assume that the structural change in demand would not have occurred but that gasoline prices and income would have developed as they did. The simulated demand is shown in figure 2, together with actual demand. Figure 2 also shows

![Figure 3. The Effect of the Driving Ban on the Demand Function for Gasoline in Mexico City, 1983–92](image)

Note: The surfaces show the demand function estimated before regulation and how it is changed by the regulation.

Source: Authors' calculations.
a 95 percent confidence interval for simulated demand without the regulation. The simulation indicates that, if the ban on driving had not subjected the demand system to a structural shift at the end of 1989, demand would have been lower in all but the first quarter following passage of the regulation. Furthermore, actual demand is outside the confidence interval for all but one observation.2

III. Household Behavior: A Vehicle Ownership Model

We now investigate how regulation could have increased gasoline consumption. Although it is possible that the regulation has increased use per car, a more likely explanation is that regulation has provoked additional car purchases in the metropolitan area because each car implicitly comes with four workday driving permits. One way of explaining our results, therefore, is that many vehicle-owning households both wanted and were able to purchase an additional vehicle when part of the service flow from each vehicle was effectively expropriated. To examine this hypothesis, we analyze data from a general-purpose household expenditure survey from 1989 (INEGI 1989). The survey was conducted before the regulation, and we use it to study the socioeconomic determinants of vehicle ownership. We adopt a discrete choice model of car ownership with household characteristics and socioeconomic variables as determinants and use household data from Mexico City to estimate the model parameters.

Model

Households must allocate their scarce resources across durable goods, non-durable goods, and savings. Let us assume a durable good (such as a car) is owned because of the value of the service flow it offers and that households behave optimally given their preferences, constraints, and resources. Then, for all households owning a car, the net value of the service flow, after subtracting short-term variable costs, exceeds the fixed costs of owning the car.

We concentrate on the household’s decision to allocate income between car services and other goods and services. Each household’s ownership decision depends on characteristics determining its desire for car services and its income. For example, we expect a car to be more useful to households with more than with fewer people due to economies of scale in using the car’s capacity. At the same time, however, for a given household income, more individuals may make a car less affordable. We also expect the demand for cars to rise with wages because higher wages increase the value of the time-saving services of a car. Ben-Akiva and Lerman (1985) discuss the assumptions underlying discrete choice modeling of car ownership and travel mode.

2. In another simulation, we use univariate forecasts of price and income in the demand model. When taken individually, price looks like white noise, and income is stationary around a positive trend. We forecast price using only its mean and forecast income using the estimated trend. The results yield even larger estimates of demand increases due to the policy.
As distant analysts we observe the household’s choices and a partial list of the household characteristics that could be associated with these choices. We proceed in two steps. First, we identify which characteristics determine how many cars a household decides to own. Second, we use this understanding to predict how their choice would change when the service flow from each car is restricted.

We assume that the household maximizes a household utility function subject to a budget constraint:

\[ U(\text{TCS}_i, \text{OC}_i) = i = 1, 2, \ldots, m \]

\[ I_i = p_c D_i + p_o \text{OC}_i \]

where \( U \) is the utility function for household \( i \), \( \text{TCS}_i \) is the transportation services that household \( i \) obtains from its cars, \( D_i, \text{OC}_i \) is the consumption of all other goods and services, \( I_i \) is the total expenditure of household \( i \), \( p_c \) is the annualized cost of owning and using a car, and \( p_o \) is the price of all other goods and services. Prices in this cross-section of households from Mexico City are assumed to be uniform.

We restrict the choices to three: no car, one car, and two or more cars. This simplification is consistent with our data because only 2 percent of 1,037 households possess more than two cars. We assume that the value of the service flow that household \( i \) obtains from owning \( j \) cars, \( \text{TCS}_{ij} \), is a function of the characteristics of the household:

\[ \text{TCS}_{ij} = f_j(z_i) = 0, 1, 2 \]

where \( z_i \) is a vector of household characteristics, and \( f_j(z_i) \) allows differences among households in the utility gained from the services of \( j \) cars. A household would presumably choose to own one car only if possessing more cars or no car both would make it worse off.

We allow for the possibility that a household’s decision to own a car also depends on unobserved variables. The observed variables are determinants only of the probability of a household owning \( j \) cars, because some of the household’s unobserved characteristics may lead it to choose a different number of cars. We assume that on average the effects of these unobserved characteristics add up to 0. Hence, the assumption that the observed choice, \( y_i = j \), is optimal implies that the probability of household \( i \) choosing \( j \) vehicles is the probability that its total utility is maximized by owning \( j \) vehicles.\(^3\)

\[ \text{Prob}(y_i = j) = \text{Prob}(U_i(j) > U_i(k)) \quad k, j = 0, 1, 2, \quad k \neq j \quad i = 1, 2, \ldots, m \]

where \( k, j = 2 \) denotes two or more cars.

**Results**

The probability of car ownership in equation 5 can be expressed in terms of observables once we assume that household utilities are linear in their argu-

\(^3\) This is a joint probability distribution: \( U(j) > U(i) \), and \( U(j) > U(k), j \neq k, j \neq i. \) See Manski and McFadden (1981).
ments and that errors have a log Weibull distribution (see appendix B for the derivation).

\[
\text{Prob}(y_i = j) = \exp(X_i \beta_j) / \left( \sum_{k=0}^{2} \exp(X_i \beta_k) \right), j = 0, 1, 2 \quad i = 0, 1, \ldots, m
\]

where \(X_i\) includes household characteristics as well as total expenditure, which we may interpret as a proxy for disposable income. Household characteristics and total expenditure feed into the utility functions through the coefficients \(\beta_0, \beta_1, \text{ and } \beta_2\). For example, the vector \(\beta_1\) tells us the importance of each of the household characteristics and income in determining the value of one car, relative to none, and \(\beta_2\), for two cars relative to none. If \(X_i \beta_1\) is greater than \(X_i \beta_2\) and \(X_i \beta_0\), household \(i\) would choose to own one car. The coefficients indicate how the variable increases the probability of having \(j\) cars, as opposed to having no car.

We estimate the parameters of the model by maximizing the multinomial logit likelihood function that is defined in appendix B. We use a two-step procedure: first maximizing the likelihood function with respect to all variables and subsequently reestimating the model using only the variables that were significant in the first step. The results are given in table B-1. The signs of the coefficients are plausible a priori. The more children a household has, or the more members who have higher education, the greater their preference to have a car, given similar incomes. The importance of education and average wages is higher for the second car than for the first. For a cross-section, discrete choice model, the fit is reasonably good. We compare actual with predicted outcomes in table B-2. Out of the 694 households that do not own cars, the model predicts “no car” correctly for 94 percent, but it predicts car ownership correctly only for 43 percent of the car-owning households.

**Simulation of a Reduction in the Service Flow from Each Car**

Next we use our model of car ownership to examine the likely response to a ban on driving. We model the ban as reducing the service flow a household gets from each car it owns. Once the value of the car's service flow is restricted, a household’s optimization problem has changed. Poor households for whom the value of the service flow initially only marginally exceeds the costs of car ownership might want to sell their car after the ban reduces its service flow. Wealthier households might find a second car justified because it can substitute for the expropriated service flow and perhaps provide additional services.

We model the utility of having a car under the regulation by applying a restriction factor, \(\alpha\), to the utility of the service flow: \(U'(TCS_1, OC_1) = U(\alpha TCS_1, OC_1)\). But first we need to make assumptions about the utility of having one car, \(U(TCS_1, OC_1)\). We assume that having one car is additive in its

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4. We assume, in these calculations, that prices do not change, including those of used cars. In Mexico data on used car values are used for insurance purposes and reflect standard depreciation factors rather than market conditions.
two arguments and that there is no constant term in the part corresponding to travel services. Restricting this constant to 0 means that a household—independently of income—does not draw any utility from a car if there are no adults and no children (thus no education or wages). A restrictive assumption is required to simulate the effect of the ban, and we believe this one is reasonable. The approach is vulnerable to a “Lucas critique,” because we assume that we know how the determinants of car ownership are changed by the regulation, that is, how parameters are changed. We perform some sensitivity analysis and discuss potential weaknesses in the subsequent sections.

The following equations display the model of the utility of household car ownership.

\[ U(TCS_i, OC_i) = V(TCS_i) + W(OC_i) \]

where

\[ V(TCS_i) = \beta_{11}\text{Child} + \beta_{12}\text{Adult} + \beta_{13}\text{MedEd} + \beta_{14}\text{HighEd} + \beta_{15}\text{WagePW}, \]

\[ W(OC_i) = \beta_{10} + \beta_{16}\text{TotExp}. \]

where our household characteristics are number of children in the household (Child), number of adults (Adult), number of people with high-level and inter-mediary-level education (HighEd and MedEd, respectively), and average wages earned by the wage earners in the household (WagePW). The vector of exogenous variables includes these household variables plus a constant (C), and total expenditure (TotExp), our proxy for disposable income. The assumptions imply that income does not influence the utility of car services directly but that it does so indirectly through the shadow price of income available for other consumption. With a negative constant and a positive income coefficient in \( W(OC_i) \), car ownership becomes more likely as the total budget grows, because higher income allows for more other expenditures if allowance is made for the costs of owning and operating a vehicle.

We simulate how the utility of owning one car is changed by the restriction as follows. We take the estimated utility function as given and simulate the car usage restriction by applying a restriction parameter \( \alpha \) to the value of the service flow from the car, that is, to parameters \( \beta_{11} \) through \( \beta_{15} \):

\[ U'(TCS_i, OC_i) = \beta_{10} + \beta_{16}\text{TotExp} + \alpha \sum_{i=1}^{5} \beta_{1i} x_i \]

where \( U' \) is the simulated utility under the restriction.

The value of \( \alpha \) is subject to sensitivity analysis. For households with one car, if travel days have no substitutability and the car is used only on workdays, \( \alpha \) is 4/5; if the car is used approximately evenly across the week and the weekend, then \( \alpha \) would be 6/7. If a household can comply with the restriction without losing any service value (say, it uses the car for some trips every week, but the trips can be moved from one day to another without any costs) then \( \alpha \) would be 1. At the other extreme, if the household needs the car only on the day the restriction is binding, \( \alpha \) would be 0. This latter case is unlikely, given that the car
registry process as well as the car market allows owners to influence which week-
day is banned.

For households that own more than one car, we assume that one car can
substitute for the other on restricted days, so the restriction has little or no ef-
fect. If the household has only one driver, this would be accurate. For a two-
driver household, the regulation cuts the service flow from two cars on five
workdays to two cars on three workdays and one car on two workdays. Thus, if
a two-car household would otherwise use its second car only three days a week,
this assumption is accurate. Otherwise, it is an approximation, and it is wrong if
households with two cars have the same difficulty managing without every car
on each workday as has a one-car household. Our modeling assumptions should
be interpreted as the effective reduction in the value of the service flow for a one-
car household in comparison with a multicar household. Finally, we assume
that households without cars would not change their behavior, because optimi-
zation theory predicts that an added constraint can change the optimal choice
only if it restricts the originally optimal choice.

Results for different restriction coefficients are given in table 2. The simula-
tions produce an optimal number of cars for each household before and under
the regulation. In table 2 we report as sellers the households that the model
predicts will switch from being a one-car household to being a no-car household
and as buyers the households that will have one car in the preregulation regime,
but two under regulation.

The model indicates that, for restriction factors in the range of 0.8 to 0.9, the
number of “sellers” exceeds the number of “buyers” by 2 to 3 percentage points.
Thus, the model predicts a slight increase in exports of used cars to the rest of
the country as a result of the restriction. However, most observers believe that
the opposite occurred. Many Mexico City households have bought an addi-
tional car in response to the regulation. Increased purchases of used cars is con-
sistent with our estimation that total gasoline consumption has increased, but
this result is not indicated by our car ownership model.

Figure 4 illustrates the main features and results of the car ownership model.
The first graph in figure 4 is constructed by projecting the model to a two-

Table 2. Simulating Car Ownership under the Regulation
(percentage of predicted stock of cars)

<table>
<thead>
<tr>
<th>Restriction coefficient, $\alpha$</th>
<th>Sellers</th>
<th>Buyers</th>
<th>Net purchases</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.95</td>
<td>5</td>
<td>3</td>
<td>-2</td>
</tr>
<tr>
<td>0.90</td>
<td>8</td>
<td>6</td>
<td>-2</td>
</tr>
<tr>
<td>0.85</td>
<td>11</td>
<td>8</td>
<td>-3</td>
</tr>
<tr>
<td>0.80</td>
<td>14</td>
<td>12</td>
<td>-2</td>
</tr>
<tr>
<td>0.75</td>
<td>18</td>
<td>16</td>
<td>-2</td>
</tr>
</tbody>
</table>

Note: Sellers are households that the model predicts will switch from one car in the preregulation
regime to no car under the regulation. Buyers are households that the model predicts will switch from
one car to two cars.

Source: Authors’ calculations based on INEGI 1989 household survey, Mexico City subsample.
Figure 4. *Income Distribution and the Utility of Car Ownership in Mexico City*

**Utility of car ownership**

- Preregulation household does not own a car, $U_0$
- Preregulation household owns one car, $U_1$
- Preregulation household owns two cars, $U_2$
- Under the regulation, household owns one car, $U_1'$

**Income distribution of the households surveyed**

**Note:** The curves are drawn as follows: $U_0$ is only a reference—the horizontal line. For $U_1$, we use the estimated one-car coefficients, plugging in the variables for each household in the data set to calculate 1,037 utility levels given that the household has one car. Next, we use a univariate ordinary least squares model to regress these utility levels on constant and total expenditures. For $U_1'$, we follow the same procedure, but with the two-car coefficients $U_1'$, utility as a function of income given one car under the restriction, is calculated using a shift parameter $\alpha$ of 0.8 (see text).

**Source:** Authors' calculations.
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dimensional one: utility as a function of income given that the household owns no, one, or two cars. For each household a separate utility level is calculated under three different preregulation scenarios: the household owns no car ($U_0$), one car ($U_1$), or two or more cars ($U_2$). We plot these utilities against the household's income level. The figure shows that each of the conditional utility functions $U_0$, $U_1$, and $U_2$ has an income range for which it gives the highest utility. This is the income range for which that specific number of cars is optimal when there is no regulation. For the lowest income range, $U_0$ is highest, so no car is the optimal choice for households in that range. As income increases, the utility of having one car increases (the slope of $U_0$ is normalized to 0). In the income range to the right of the intersection of $U_0$ and $U_1$, households typically own one car. At an even higher income range, households typically own two or more cars.

$U_1'$ shows the utility of having one car after the restriction is imposed, using a restriction factor of 80 percent (that is, 80 percent of the service flow remains). $U_1'$ is lower than $U_1$: we thus have a reduction in the size of the income range for which one car is the optimal choice. For incomes around Mex$800,000, the optimal number of cars has shifted from one to zero. These households, according to our simple model, would sell their car. But households earning about Mex$2.8 million would respond by expanding vehicle ownership from one to two (or more). Thus our model predicts that the income threshold a household must pass to buy its first car moves upward, while the income threshold for buying a second car moves downward. The second graph in figure 4 depicts the income distribution of the households surveyed. There is a greater density of households in the range of sellers than in the range of buyers, but the latter range is larger and is supported by greater per household incomes.

We may conclude that our simple ownership model is successful in demonstrating a range of buyers and sellers in similar magnitude, but it does not explain increased vehicle usage, because it does not point to increased ownership. Are there simplifying assumptions in our model that could plausibly cause an underestimation of net purchases? Two are worth considering: used car prices and sunk costs. In our model, used car prices are assumed to be unaffected by the ban. When part of the service flow from cars is expropriated, ceteris paribus the value of used cars should fall. However, under the ban a car also becomes a bundle of implicit driving permits, which should increase its price. Therefore the net effect of the regulation on (used) car prices cannot be known a priori. This does not mean that car prices have not risen or fallen in reality, as our model assumes. We do not know how to adjust used car prices to correct this weakness in the model because price variation is absent in our data.

A related problem is that when we abstract from the sunk cost aspect of investing in a car, we introduce an asymmetry if a regulatory change reduces the service flow from a car. Some owners who would want to sell in the absence of sunk costs will hesitate and not sell when there are sunk costs. No correspond-
ing hesitation applies to the households induced to purchase an additional car. This is thus a weakness that could also explain why our model, which ignores sunk costs, can underestimate the number of net purchases.\(^5\) Our analysis is overly simplistic in other ways, however, as in lumping all multicar households into a single category and in making assumptions about the specific way utility function parameters are changed by the regulation.

**Supporting Data: What Happened to Vehicle Ownership?**

We have recently obtained aggregate annual data that shed light on the effects of the ban on vehicle ownership. Table 3 shows data on sales of new vehicles and increases in the number of vehicles registered in Mexico City (the Federal District) and the rest of the country, averaged over seven preregulation years and four regulation years.

An area's net import of used vehicles is inferred by subtracting the sales of new vehicles from the net increase in vehicles registered (assuming vehicle scrapage is 0). Before regulation capital city households were consistent net exporters of used vehicles to the rest of the country—about 74,000 vehicles per year. Such a flow is to be expected. Given that city households on average have higher incomes, poorer regions buy used vehicles from the capital. In Mexico City, the tradition has been that visitors from far away gather at the Aztec Stadium to bargain for used cars.

Table 3 also shows that Mexico City's traditional exports of used vehicles turned to net imports under the regulation. During the years of regulation, Mexico City has been importing an average of 85,000 vehicles per year from the rest of the country. A statistical test does not confirm that a break exists at the introduction of the regulation. This is not surprising given anomalies in the data. For example, 300,000 vehicles—15–20 percent of the stock—appear to vanish in 1986, resurfacing in 1988.

Expansion of car ownership through used cars has obvious implications. Older cars are typically less fuel efficient due to both initial design and deterioration. Part of the poorer fuel economy of older cars is due to incomplete combustion, leading older cars to be more polluting per kilometer and per liter of fuel consumption. Finally, the fact that the regulation artificially ties up and idles capital in the wrong places implies that it is costly to the nation.

We include other data in table 3 that can be disaggregated by area. If phone calls and electricity consumption are good proxies for personal incomes, then they indicate that there was no change in personal incomes in Mexico City and the rest of the country that would explain the observed turn in the flow of used cars. Finally, table 3 indicates that ridership of the subway system in Mexico City has declined in the years of regulation. This could

\(^5\) These effects are explored in option pricing theory (not buying a car leaves the option of buying one later), and this particular effect is called **hysteresis** (Dixit and Pindyck 1993: 136). In a market for used cars, transactions costs may be high due to asymmetric information about quality, giving a theoretical underpinning for the existence of sunk costs (Akerlof 1970).
Table 3. Supporting Data for the Household Car Ownership, Mexico City, 1983–93
(annual average)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sales of new vehicles (thousands)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mexico City (federal district)</td>
<td>80</td>
<td>154</td>
</tr>
<tr>
<td>Rest of Mexico</td>
<td>127</td>
<td>237</td>
</tr>
<tr>
<td><strong>Increase in vehicles registered (thousands)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mexico City (federal district)</td>
<td>7</td>
<td>239</td>
</tr>
<tr>
<td>Rest of Mexico</td>
<td>174</td>
<td>250</td>
</tr>
<tr>
<td><strong>Net import of vehicles (thousands)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mexico City (federal district)</td>
<td>-74</td>
<td>85</td>
</tr>
<tr>
<td>Rest of Mexico</td>
<td>47</td>
<td>13</td>
</tr>
<tr>
<td><strong>International phone calls (percentage growth)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mexico City</td>
<td>27.2</td>
<td>24.1</td>
</tr>
<tr>
<td>Whole country</td>
<td>19.5</td>
<td>29.7</td>
</tr>
<tr>
<td><strong>Local phone calls (percentage growth)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mexico City</td>
<td>12.9</td>
<td>12.9</td>
</tr>
<tr>
<td>Whole country</td>
<td>19.5</td>
<td>29.6</td>
</tr>
<tr>
<td><strong>Electricity consumption (percentage growth)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mexico City</td>
<td>7.0</td>
<td>7.0</td>
</tr>
<tr>
<td>Whole country</td>
<td>5.0</td>
<td>3.0</td>
</tr>
<tr>
<td><strong>Metro ridership (percentage growth)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mexico City</td>
<td>5.7</td>
<td>-2.4</td>
</tr>
</tbody>
</table>

*Note:* We analyze the vehicle data using two different definitions of Mexico City, the federal district and the federal district plus the state of Mexico. The regulation applies to the Mexico City Metropolitan Area (MCMA), comprising the federal district plus part of the state of Mexico. The conclusions are very similar. We show only the data from the federal district here. For other data, the area covered by Mexico City varies. The area for electricity consumption is larger still than the MCMA. We use a long data series (1983–93) due to the noisiness of registration data, but this does not affect the conclusions.

*Source:* Mexico’s Automobile Manufacturers Association, Banco de México, and INEGI.

be interpreted as another indication that the regulation does not work according to intentions. It could also provide a partial explanation: if the subway and other public transport systems have little capacity to serve additional passengers, this would contribute to the choice of additional vehicles as a compliance strategy.

In this section we have described and estimated a theoretical model of car ownership showing how a regulation creates buyers as well as sellers; we have also considered other data supporting the hypothesis that households in Mexico City have responded to the driving ban by buying additional used cars from the rest of the country.
IV. CONGESTION AND WEEKEND TRAVEL

While the above explanation explores possible increases in vehicle ownership, this section explores features that could break or reduce the presumed effect on driving per car. The ownership model assumes that the regulation reduces the service flow from each car, other things being equal. In fact, as the service flow from other cars is reduced as well, congestion levels could decline and travel speeds could increase, making each car more useful. To explore this effect, let demand, $\tau^d$ (we now represent the vehicle and its services with one variable, abstracting from the distinction between ownership and use) consist of an exogenous component, $k$, and a component sensitive to travel time, $t$ (say, the average time that it takes an individual to drive to work, given the congestion levels), $\tau^d = k + \nu(t)$. Further, to describe the road network’s capacity, let travel time be a function of demand, $t = t(\nu)$. If the regulation reduces the exogenous component of demand, $k$, the equilibrium effect will be dampened by a rebound in demand due to increased speed (appendix C), as described by equation 9.

\[
\frac{d\nu}{dk} = \frac{1}{1 - \varepsilon^{d}_{\tau, t} \varepsilon^{t}_{\tau, \nu}}.
\]

The first elasticity (travel demand with respect to time) is negative, and the second (the road network’s supply of travel time, with respect to additional entering vehicles) is positive. Thus the denominator is at least 1, dampening any direct effect that a regulation has on travel demand. We can see that if either (or both) of the elasticities is 0, the equilibrium effect on demand is the direct effect: a car removed from the streets on Tuesday simply reduces overall traffic on Tuesday by one car. This will in fact be the case on completely uncongested roads, but it is not realistic for workday conditions. By contrast, if the product of the two elasticities multiplied by each other is $-1$ or $-3$, then the equilibrium reduction after reducing traffic by, say, four cars on Tuesday is only two cars or one car, reflecting that other vehicles enter the roads to take advantage of the reduced congestion.

Using plausible parameters we find that the multiplied elasticities could be larger than 1 in absolute value, so that the equilibrium reduction in workday traffic could be less than half of the initial reduction, due to a resurgence in speed-sensitive travel. For supply conditions, only a few estimates exist in the literature of how travel times (or speeds) respond to additional vehicles entering the road, and none exists for Mexico City. In severely congested conditions the elasticity is greater than 1, meaning that an additional vehicle reduces the total throughput of a road link. According to Small (1992), an elasticity of 2.5 reflects conditions in the middle of a range. The demand elasticity with respect to travel time savings under plausible assumptions is three-quarters of the demand elasticity with respect to gasoline prices (appendix C), or $0.75 \cdot (-0.8) = -0.6$,
using a conservative elasticity estimate of -0.8 from Mexico.\footnote{Eskeland and Feyzioglu (1997) and Berndt and Botero (1985), both with pooled estimate results for Mexico, find long-term estimates in the range of -0.7 to -1.25.} Using these values, a plausible estimate of the equilibrium reduction in traffic on a weekday is 0.40, or two cars for every five initially removed.\footnote{\[ \frac{dv}{dk} = \frac{1}{(1 - e^{-\rho t} e^{\lambda t})} = \frac{1}{(1 - (-0.6) \times 2.5)} = 1/2.5 = 0.4. \]}

But how could a reduction in traffic on weekdays—even if only 40 percent of the initial reduction—result in an increase in total traffic? Another unmodeled effect is the distinction between weekdays and weekends. Trips on different days will generally be imperfect substitutes, and less congestion and absence of regulation are only part of what makes weekend trips special. But if the equilibrium reduction in traffic is only 0.4 cars on a weekday when one car is initially removed from the road by the regulation and if as much as 40 percent of the cars originally removed from the street make an additional trip on the weekend to compensate, Mexico City could see total travel increase even if no additional cars are purchased. In this case much of the intended effect of the regulation might be undermined by increased weekend driving, and the additional car purchases then lead to a significant increase in car use.

As attractive as the substitution of weekend for weekday driving hypothesis appears, we are unable to confirm it in empirical tests. We used daily observations of carbon monoxide pollution data to provide a daily proxy for driving, because the link to emissions from cars is direct and quick. Using weekly observations of the ratio of the pollution concentration in the weekend to that during weekdays, we find two results (see Baynham 1997). First, regressing the ratio on a regulatory dummy alone, the regulation has a significant effect, increasing the average ratio by 7 percentage points, from 86 to 93 percent. Second, once additional variables, such as a time trend, are included, the effect becomes insignificant. Thus the effect of the regulation on the relative importance of weekend driving is small if existent at all. It is rejected by a simple statistical test.

So although it is conceivable that net weekday reductions are considerably less than direct weekday reductions and that weekend travel increased, our analysis of daily pollution data does not give firm indication of any major change in the relative importance of weekend travel. This leaves induced purchases of additional vehicles as the apparent cause of the counterproductive results of the regulation.

V. SUMMARY AND CONCLUSIONS

We estimated a gasoline demand function based on aggregate time-series data to analyze the effect of the driving ban in Mexico City. Surprisingly, our results indicated that total car use in Mexico City has been increased by the regulation. We focused on additional car purchases in the metropolitan area as the most likely explanation of increased driving, because households with unchanged car use.
ownership would be unlikely to increase driving. Supporting data on vehicle ownership supported this interpretation. In preregulation years, Mexico City on average exported 74,000 used vehicles annually to the rest of the country. In the first four years of the regulation, Mexico City imported 85,000 vehicles annually.

Assuming that car ownership is motivated by the service flow that cars offer, we estimated a model of household ownership decisions and simulated individual responses to the ban. Our model indicated that although some households would want to buy more cars as a result of the regulation, a somewhat greater number of households would want to reduce their car ownership. An assumption—that cars are bought and sold without transaction costs—could be a reason why the car ownership model would overestimate the number of households that would sell their car.

We also noted additional features excluded in the ownership model. First, congestion-sensitive demand for travel would imply that net reductions in travel on weekdays might be significantly less than direct reductions. Second, some suppressed weekday trips could show up as additional weekend trips. Third, travel may have shifted toward less fuel-efficient old cars so that aggregate gasoline consumption—and pollution—could increase even if travel were constant or slightly reduced.

There is thus ample evidence that the ban imposed high compliance costs for households, much higher than those of alternative market-based policies such as gasoline taxes. Moreover, many individuals chose a compliance strategy that led to no reductions—or even increases—in car use and acquired a used car with lower technical standards. Thus results for accidents and pollution could be worse than what is indicated for total gasoline consumption and congestion.

Finally, we should mention that we have not analyzed how regulation and taxation would differ in terms of how costs are distributed. Such differences may exist—it is unclear whether they would favor one strategy—but are hardly relevant when a costly strategy is found to be counterproductive.

**Appendix A. Time-Series Properties and Estimation Results of the Aggregate Data**

*Time Series Properties*

The autocorrelation function and the augmented Dickey-Fuller unit root tests reject nonstationarity for all but the income data. The autocorrelation function rejects nonstationarity for income, but the augmented Dickey-Fuller does not. Therefore, for the income series we perform further tests. Income in itself is not the focal point, but rather a factor that affects consumption. We therefore run a regression with consumption, price, and income. Consumption and price are stationary; therefore, the residual has the same stationarity properties as the price. Our tests show that the residual is stationary. We therefore conclude that,
Table A-1. Regression of Aggregate Gasoline Consumption

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard error</th>
<th>1-tail significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-7.27</td>
<td>0.37</td>
<td>0.0000</td>
</tr>
<tr>
<td>ln(price)</td>
<td>-0.17</td>
<td>0.06</td>
<td>0.0167</td>
</tr>
<tr>
<td>ln(income)</td>
<td>0.06</td>
<td>0.02</td>
<td>0.0050</td>
</tr>
<tr>
<td>Dummy*</td>
<td>-1.93</td>
<td>0.50</td>
<td>0.0060</td>
</tr>
<tr>
<td>Dummy**ln(price)</td>
<td>0.12</td>
<td>0.11</td>
<td>0.2820</td>
</tr>
<tr>
<td>Dummy**ln(income)</td>
<td>0.18</td>
<td>0.07</td>
<td>0.0092</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.942</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson statistic</td>
<td>2.28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F-statistic</td>
<td>115</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability F-statistic</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The dependent variable is ln(total gasoline consumption).
a. The value of the dummy variable is 0 for 1984-89 and 1 for 1990-92.
Source: Authors’ calculations.

for our purposes, it is sufficient to model income as a stationary variable, and ordinary least squares estimation is appropriate.

Regression Results

We estimated equation 1 using ordinary least squares, and table A-1 reports the estimation results. The proxy for income in Mexico City is the number of international (outgoing) telephone calls. Others variables, such as quarterly figures for gross national product, local industrial output, and local electricity consumption, performed less well statistically, judged in terms of a preregulation demand model. The changes in the constant and the income elasticity are statistically significant, while the changes in the price elasticity are not.

APPENDIX B. DERIVATION AND ESTIMATION OF THE CAR OWNERSHIP MODEL

Derivation

First, we combine the budget constraint and the definition of total transportation services from cars with the utility function by substituting equations 3 and 4 into equation 2:

\[(B-1) \quad U_{ji} = U[f_i(z_j), (I_i - p_c D_i)/p_o]\]

where, \(U_{ji}\) is the utility of household \(i\) if it has \(j\) cars, \(f_i(z_j)\) is a vector of household characteristics, \(I_i\) is the total expenditure of household \(i\), \(p_c\) is the annualized cost of owning and using a car, \(D_i\) is the number of cars owned by household \(i\), and \(p_o\) is the price of all other goods and services. Second, we assume that the utility function is linear in its arguments:

\[(B-2) \quad U_{ji} = \mu_{ji} + \epsilon_{ji} \quad j = 0, 1, 2 \quad i = 1, 2, \ldots, m\]

and \(\mu_{ji} = X_i \beta_j\), where \(X_i\) includes household characteristics as well as total expenditure.
Third, we establish the probability distribution of owning \( j \) cars. For the probability of \( i \)'s not owning any vehicle, we obtain:

\[
\text{Prob}(y_i = 0) = \text{Prob}(U_{0i} > U_{ki}), \quad k \neq 0,
\]

\[
= \text{Prob}[(e_{0i} - e_{1i} > \mu_{1i} - \mu_{0i}), (e_{0i} - e_{2i} > \mu_{2i} - \mu_{0i})]
\]

where \( k \) is the number of cars owned by household \( i \).

If \( e_{ij} \) has a log Weibull distribution, the probability of choosing \( j \) vehicles is a logistic function:

\[
\text{Prob}(y_i = j) = \exp(X_i\beta_j) \left[ \sum_{k=0}^{2} \exp(X_i\beta_k) \right], \quad j = 0, 1, 2 \quad i = 0, 1, \ldots, m.
\]

This is equation 6 in section III.

For estimation, because utility is ordinal, we normalize utility to be 0 for the case of no cars, so that the model estimates the additional utility of owning a positive number of cars. The decision process is restated in terms of deviations from the utility of owning no cars:

\[
\text{Prob}(y_i = j) = \exp(X_i\beta'_j) \left[ \sum_{k=0}^{2} \exp(X_i\beta'_k) \right], \quad j = 0, 1, 2 \quad i = 0, 1, \ldots, m
\]

where \( X_i\beta'_j = X_i\beta_j - X_i\beta_0 \). The decision process about the number of cars to own by all households can be put together into a standard multinomial logit likelihood function:

\[
L = \prod_{i=1}^{m_0} P(y_i = 0) \prod_{i=m_0+1}^{m_0+m_1} P(y_i = 1) \prod_{i=m_0+m_1+1}^{m_0+m_1+m_2} P(y_i = 2)
\]

where, \( m_0, m_1, \) and \( m_2 \) indicate number of households in each category in the data set that is sorted with respect to number of vehicles owned.

Results

Results from household data are given in table B-1. The coefficient of wage income is 0.12 for one car, which means that if the average wage income of the household increases by 1, then the extra utility the household gets from owning a car as opposed to not owning one is 0.12. The corresponding coefficient for two cars is 0.19, which means that if wage income increases by 1, then the extra utility of owning two cars as opposed to one or no car is 0.07 and 0.19, respectively. Similar interpretations follow for the other coefficients.

The predictive power of the model is illustrated in table B-2.

Finally, using the model in simulation with a restriction factor of 0.8, as is used in both parts of figure 4, 33 households come out as sellers, while 38 of the one-car households want an additional car. As predicted percentages of the stock of cars among the 1,037 households indicated, purchases and sales come out as 12 and 14 percent, respectively (see table 2).
Table B-1. **Results from the Household Car Ownership Model for Mexico City**

<table>
<thead>
<tr>
<th>Variable</th>
<th>One car, $\beta_1$</th>
<th>Two or more cars, $\beta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-2.37 (0.21)</td>
<td>-4.92 (0.27)</td>
</tr>
<tr>
<td>Child</td>
<td>0.12 (0.05)</td>
<td>*</td>
</tr>
<tr>
<td>Adult</td>
<td>-0.19 (0.07)</td>
<td>*</td>
</tr>
<tr>
<td>Secondary-level education</td>
<td>0.20 (0.08)</td>
<td>*</td>
</tr>
<tr>
<td>Tertiary-level education</td>
<td>0.79 (0.11)</td>
<td>1.32 (0.12)</td>
</tr>
<tr>
<td>Wage per worker</td>
<td>0.12 (0.03)</td>
<td>0.19 (0.03)</td>
</tr>
<tr>
<td>Total expenditure</td>
<td>1.62 (0.27)</td>
<td>1.87 (0.28)</td>
</tr>
</tbody>
</table>

Note: The estimated equation is $U(j \text{ cars}) - U(\text{no car}) = X\beta + e$ where $X$ is the vector of explanatory variables. All the reported coefficients are significantly different from 0 with 95 percent confidence. Standard errors are in parentheses.

a. The coefficient is not significantly different from 0 in the first-stage estimation.

Source: Authors' calculations based on INEGI 1989 household survey, Mexico City subsample.

Table B-2. **The Predictive Power of the Household Car Ownership Model for Mexico City**

<table>
<thead>
<tr>
<th>(number of households)</th>
<th>Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No car</td>
</tr>
<tr>
<td>No car</td>
<td>654</td>
</tr>
<tr>
<td>One car</td>
<td>172</td>
</tr>
<tr>
<td>Two cars</td>
<td>25</td>
</tr>
<tr>
<td>Total</td>
<td>851</td>
</tr>
</tbody>
</table>

Source: Authors' calculations based on INEGI 1989 household survey, Mexico City subsample.

**APPENDIX C. EQUILIBRIUM CHANGE IN TRAFFIC WHEN THERE IS AN EXOGENOUS CHANGE IN DEMAND FOR TRAVEL**

How does travel respond in equilibrium if the road's capacity to supply rapid travel is declining as the level of traffic increases and if the demand for travel is sensitive to congestion levels?

Assume that demand for vehicle use, $\nu$, is the sum of two components: $f(t)$, which depends on congestion levels, represented by the time spent making a certain trip, $t$, and $k$, which is exogenously given:
(C-1) \[ v = f(t) + k. \]

Also assume that the road link’s capacity to get the vehicle through within a certain time, \( t \), depends on the number of vehicles entering the road, \( v \):

(C-2) \[ t = g(v). \]

For equilibrium, we have:

(C-3) \[ v = f[g(v)] + k. \]

(C-4) \[
\frac{dv}{dk} = \frac{\frac{\partial f}{\partial t} \frac{\partial g}{\partial v} dv + dk = >}{1 - \frac{\partial f}{\partial t} \frac{\partial g}{\partial v}}
\]

where \( \varepsilon^d \) and \( \varepsilon' \) are the demand and supply elasticities of travel time. The demand elasticity of travel with respect to travel time is negative (if travel time goes up, people are less interested in traveling), while the supply elasticity of travel time with respect to entering vehicles is positive (as more vehicles are on the road, traffic slows, and travel time for a given trip rises). Thus unless one, or both, of the elasticities is 0, the equilibrium effect on travel is less than 1, that is, less than the initial, exogenous reduction in demand. Any combination of elasticities that yields a high product in absolute value would reflect conditions close to Downs law: an exogenous reduction of congestion levels would immediately lead to an increase in the demand for travel that swamps the initial effect, so that congestion levels are back at normal. Put differently, highly congested roads can easily be normal.

More cautiously argued, what are plausible values? For supply, we can immediately establish that \( \varepsilon^d_{t,v} = 0 \) represents an extreme case of road conditions in which vehicle density is so low that an additional vehicle does not slow down other vehicles. Positive values represent natural conditions in which a positive shadow price for road capacity is natural for urban roads and intercity highways (see Hau 1992). Values greater than 1 represent heavily congested conditions in which an additional vehicle reduces the total throughput of the road link per time unit (a 1 percent increase in entering vehicles increases travel times for all vehicles by more than 1 percent).

For the elasticity of travel demand with respect to travel times, what are plausible values? The following modeling framework, focusing on vehicle travel as a timesaving alternative, may shed light on that question.

Let utility, \( u \), be defined over cars, \( c \), other goods and services, \( o \), and leisure, \( l \):

(C-5) \[ u = u(c, o, l). \]
And let the individual budget constraint be:

(C-6) \[ p_c c + o = w(L + lc - l) + I. \]

where \( p_c \) is the price of owning and using a car, the price of other goods and services is normalized to 1, \( w \) is the wage rate, \( lc \) are the time savings offered per vehicle (\( c \) is interpreted as a continuous variable; we may view this as a model of a representative consumer), \( L \) is the endowment of human capital, and \( I \) is lump-sum income.

The Lagrangian of the consumer’s maximization problem can be written:

(C-7) \[ L = u(c,o,l) - \lambda([p_c - wlc)c + o - w(L - l) - I]. \]

The first-order conditions are:

I \[ \frac{\partial u_{cl}}{\partial c} = \frac{-(p_c - wlc)}{w} \]

II \[ \frac{\partial u_{ol}}{\partial o} = \frac{-1}{w} \]

III \[ (p_c - wlc)c + o - w(L - l) = I \]

where \( u_{cl} \) and \( u_{ol} \) are marginal rates of substitution. The interpretation of the first equation is that the value of time savings justifies part of the cost of cars, and this part is subtracted from the cost, \( p_c/w \), that would otherwise be equated with the marginal rate of substitution between cars and leisure.

To study relationships between demand elasticities, differentiate the first-order conditions with respect to the time savings offered per car:

(C-8) \[ \frac{\partial u_{cl}}{\partial c} \frac{dc}{dlc} + \frac{\partial u_{cl}}{\partial o} \frac{do}{dlc} + \frac{\partial u_{cl}}{\partial l} \frac{dl}{dlc} = 1 \]

\[ \frac{\partial u_{ol}}{\partial c} \frac{dc}{dlc} + \frac{\partial u_{ol}}{\partial o} \frac{do}{dlc} + \frac{\partial u_{ol}}{\partial l} \frac{dl}{dlc} = 0 \]

\[ (p_c - wlc) \frac{dc}{dlc} + \frac{do}{dlc} + w \frac{dl}{dlc} = wc. \]

The coefficient matrix, \( A \), is:

(C-9) \[ A = \begin{bmatrix} \frac{\partial u_{cl}}{\partial c} & \frac{\partial u_{cl}}{\partial o} & \frac{\partial u_{cl}}{\partial l} \\ \frac{\partial u_{ol}}{\partial c} & \frac{\partial u_{ol}}{\partial o} & \frac{\partial u_{ol}}{\partial l} \\ p_c - wlc & 1 & w \end{bmatrix}. \]
Assuming that $A$ is nonsingular, we may use Cramer's rule to solve for:

\begin{equation}
\frac{\partial c}{\partial l^e} |A| = \begin{vmatrix}
1 & \frac{\partial u_{el}}{\partial o} & \frac{\partial u_{el}}{\partial l} \\
0 & \frac{\partial u_{ol}}{\partial o} & \frac{\partial u_{ol}}{\partial l} \\
wc & \frac{\partial u_{el}}{\partial o} & \frac{\partial u_{el}}{\partial l}
\end{vmatrix}
\end{equation}

(C-10)

Similarly, when we differentiate the first-order conditions with respect to car prices, we obtain:

\begin{equation}
A = \begin{bmatrix}
\frac{\partial c}{\partial p_e} \\
\frac{\partial o}{\partial p_e} \\
\frac{\partial }{\partial p_e} \\
\frac{\partial p_e}{\partial l} \\
\frac{\partial c}{\partial p_e}
\end{bmatrix} = \begin{bmatrix}
-1 \\
w \\
0 \\
-w \\
-c
\end{bmatrix}.
\end{equation}

(C-11)

Again using Cramer's rule, we have:

\begin{equation}
\frac{\partial c}{\partial p_e} |A| = \begin{vmatrix}
0 & \frac{\partial u_{el}}{\partial o} & \frac{\partial u_{el}}{\partial l} \\
-1 & \frac{\partial u_{ol}}{\partial o} & \frac{\partial u_{ol}}{\partial l} \\
w & \frac{\partial u_{el}}{\partial o} & \frac{\partial u_{el}}{\partial l}
\end{vmatrix}
\end{equation}

(C-12)

For matrix $A$, let $C_{kl}$ be the cofactor of row $k$, column $j$. We may write

\begin{equation}
\frac{\partial c}{\partial l^e} |A| = C_{21} + wc \cdot C_{31}
\end{equation}

(C-13)

\begin{equation}
\frac{\partial c}{\partial p_e} |A| = -\frac{1}{w} C_{21} - c \cdot C_{31}
\end{equation}

\begin{equation}
\frac{\partial c}{\partial l^e} = -w \frac{\partial c}{\partial p_e} = > \frac{\partial c}{\partial l^e} c = -\frac{\partial c}{\partial p_e} \frac{p_c w l^c}{p_c} = >
\end{equation}

Thus:

\begin{equation}
\epsilon_{c,l^e} = -\epsilon_{c,p_e} \frac{w l^c}{p_c}.
\end{equation}

(C-14)
Thus, the elasticity of car demand with respect to the time savings offered per car is $\frac{w^c}{p_c}$ times minus the elasticity of car demand with respect to car prices. Referring back to first-order condition I, we may see that $\frac{w^c}{p_c}$ is the share of time savings in justifying car purchases at the margin (the other share being the direct utility drawn from cars):

$$\frac{-u_d w + w^c}{p_c} = 1$$

(C-15)

$$\frac{\partial c}{\partial t} = \frac{\partial c}{\partial t^c} t = -\frac{\partial c}{\partial t} =$$

It remains to relate the concept of time savings to the concept of travel times. So let us note that:

(C-16)

$$\varepsilon_{c,t} = -\varepsilon_{c,t}^c = \frac{t}{t^c}$$

The conversion between the two elasticities thus is simply the ratio of travel times to time savings offered by the car. It takes positive values and is often in the range of 0.5 to 2.0. For instance, if a regular trip takes half an hour, and the alternative is a combination bus ride and walk that takes one hour, then both time savings and travel times are half an hour, and $\varepsilon_{c,t} = -\varepsilon_{c,t}^c$. However, if a car trip is 20 minutes and the alternative mode takes 30, the elasticity of travel demand with respect to travel time is double the elasticity of travel demand with respect to time savings.

REFERENCES

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