

# *Optica No Lineal de Superficies*

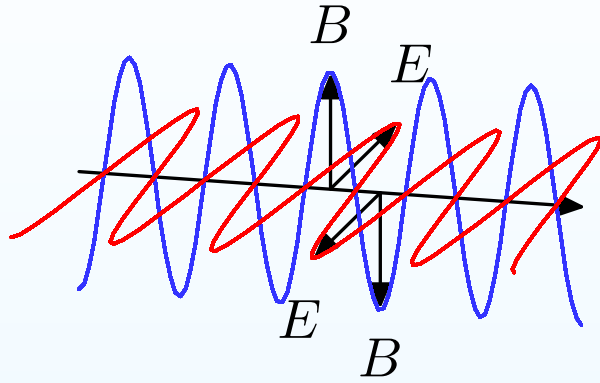
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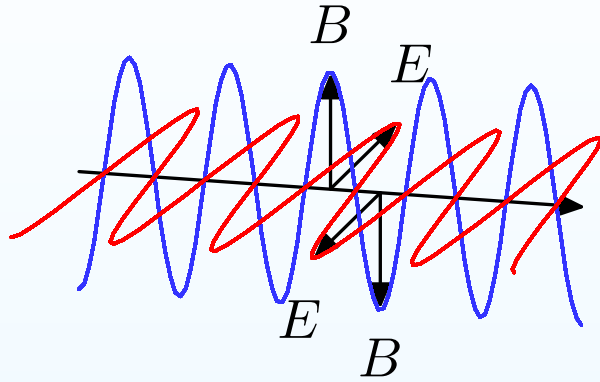
Centro de Ciencias Físicas, UNAM

# Ondas Electromagnéticas

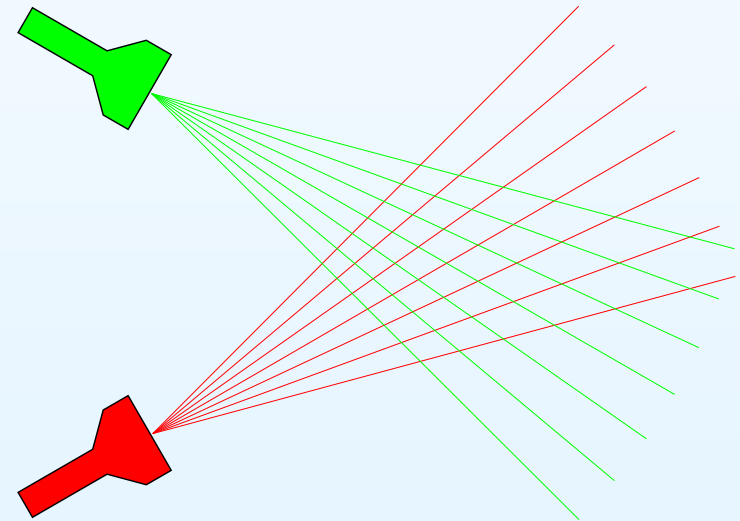
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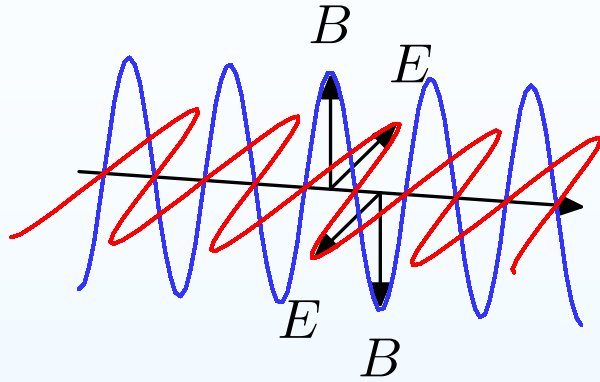
# Ondas Electromagnéticas



La luz es transparente

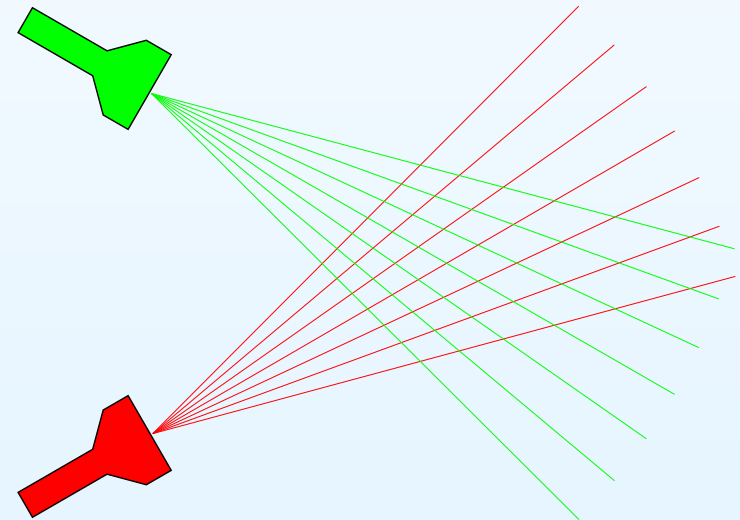


# Ondas Electromagnéticas



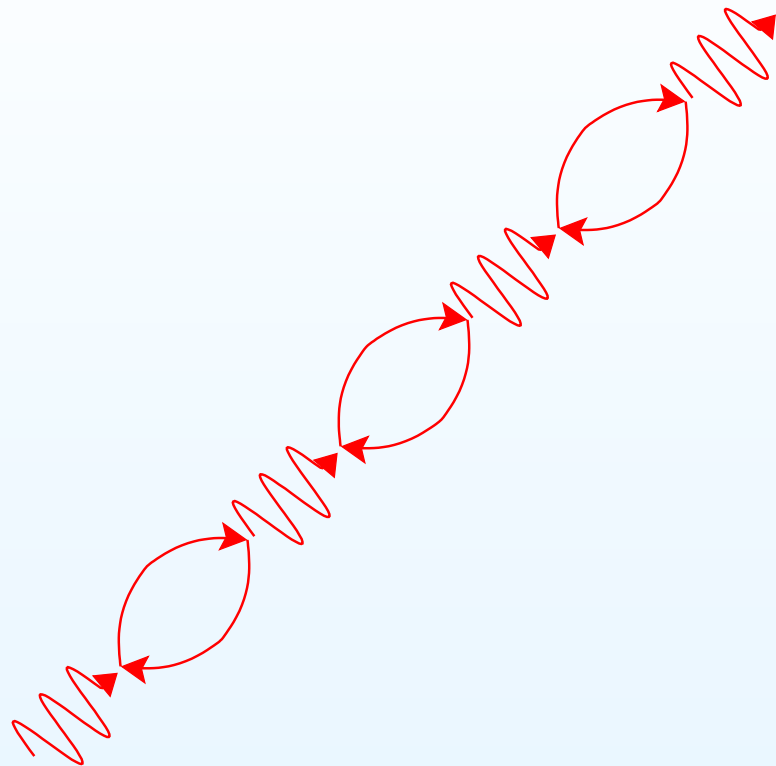
La luz es transparente

... o casi transparente.



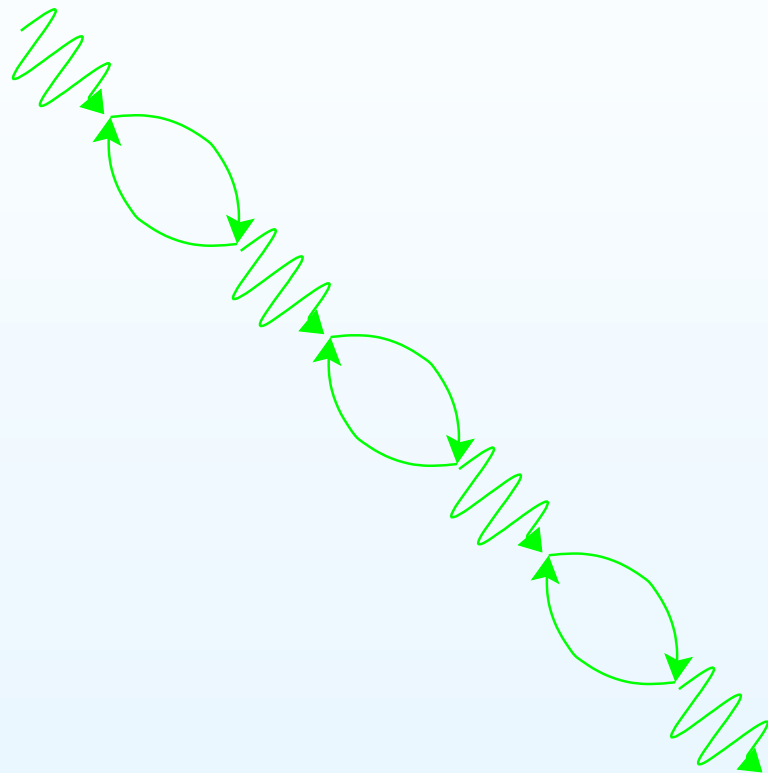
# Fotones vestidos

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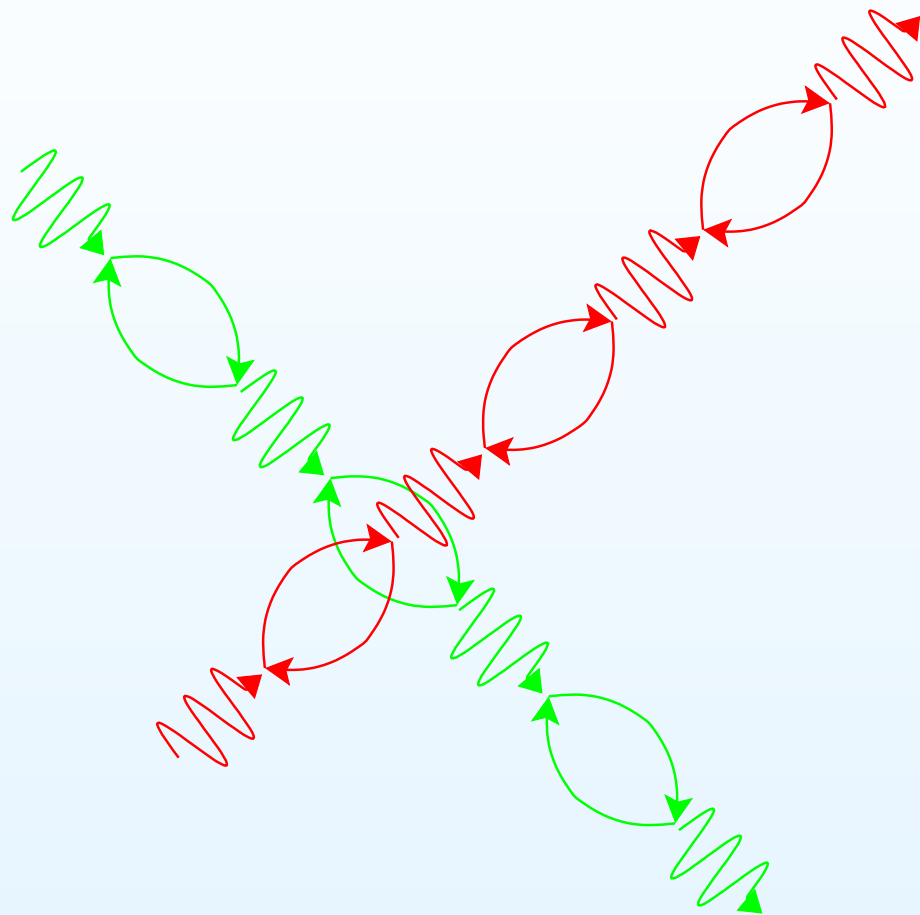
# Fotones vestidos

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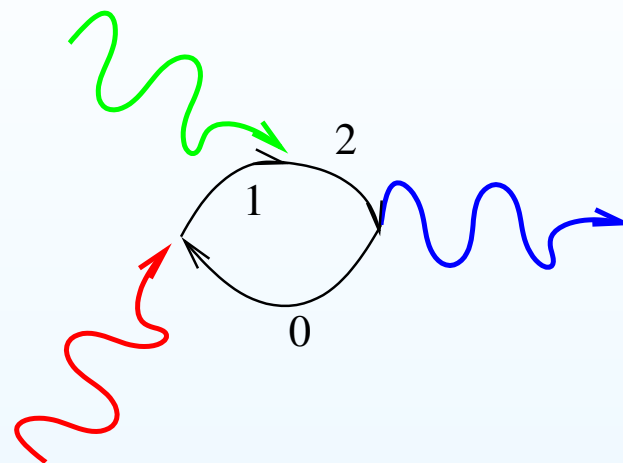
# Fotones vestidos

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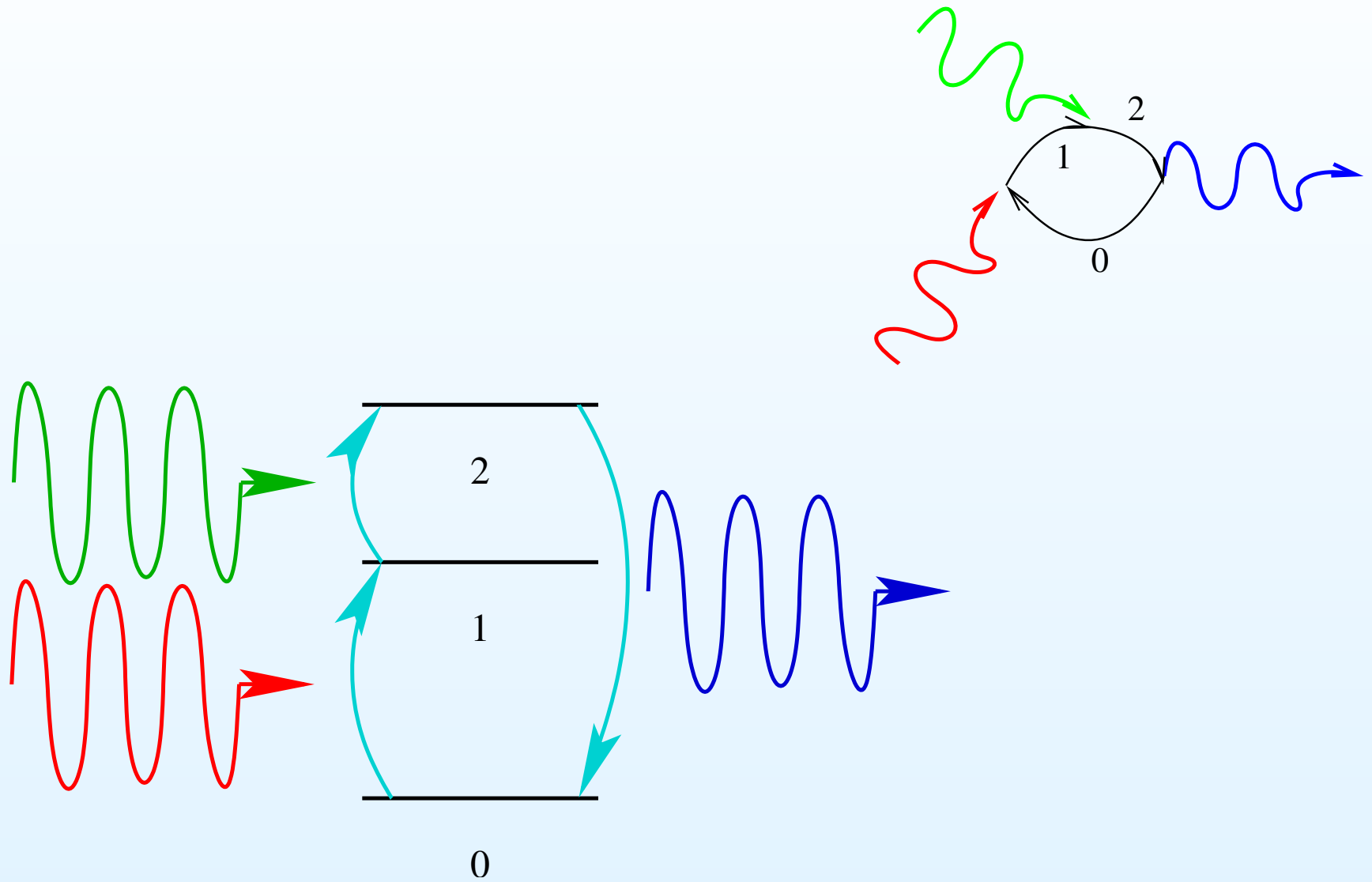
# Suma de Frecuencias

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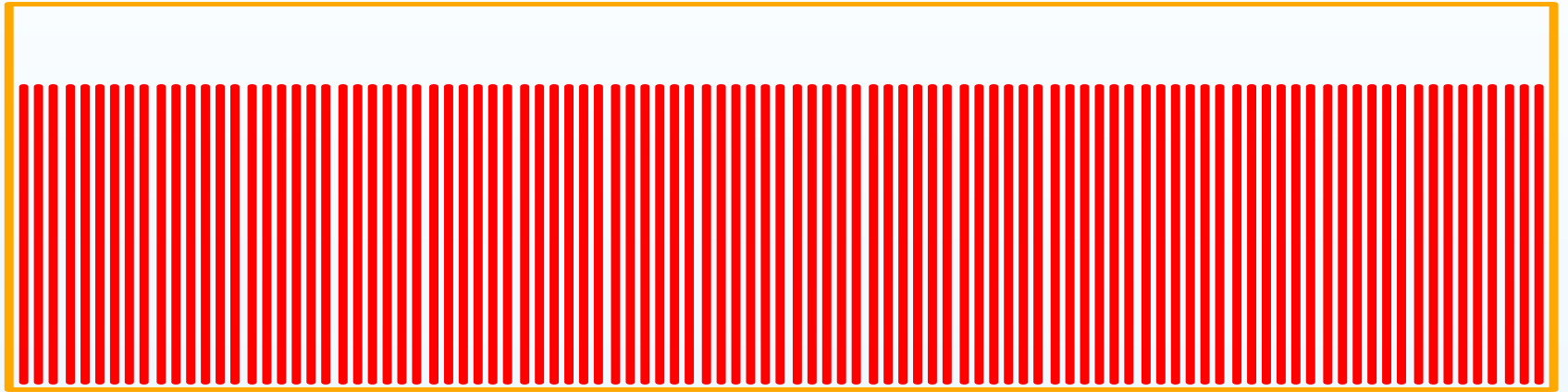


# Suma de Frecuencias



# Multiplicación de Ondas

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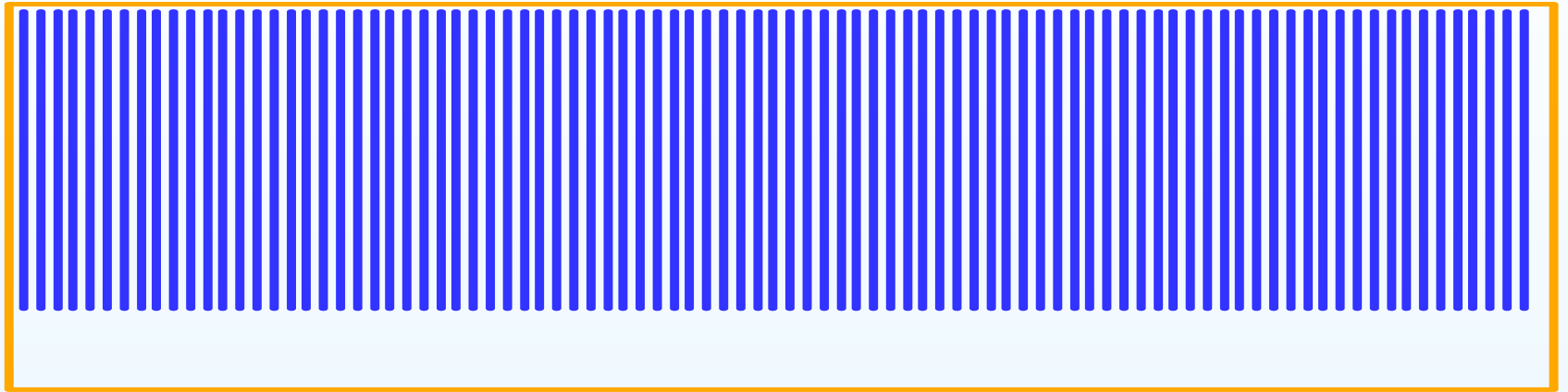


$\omega_1$

# Multiplicación de Ondas

---

$$\omega_2 = 1.1\omega_1$$

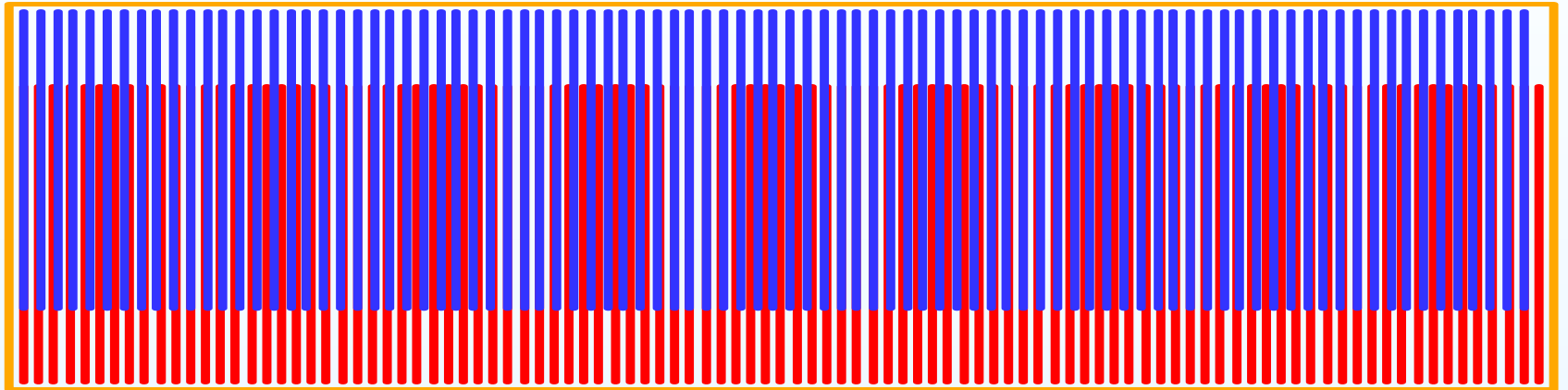


$$\omega_1$$

# Multiplicación de Ondas

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$$\omega_2 = 1.1\omega_1$$

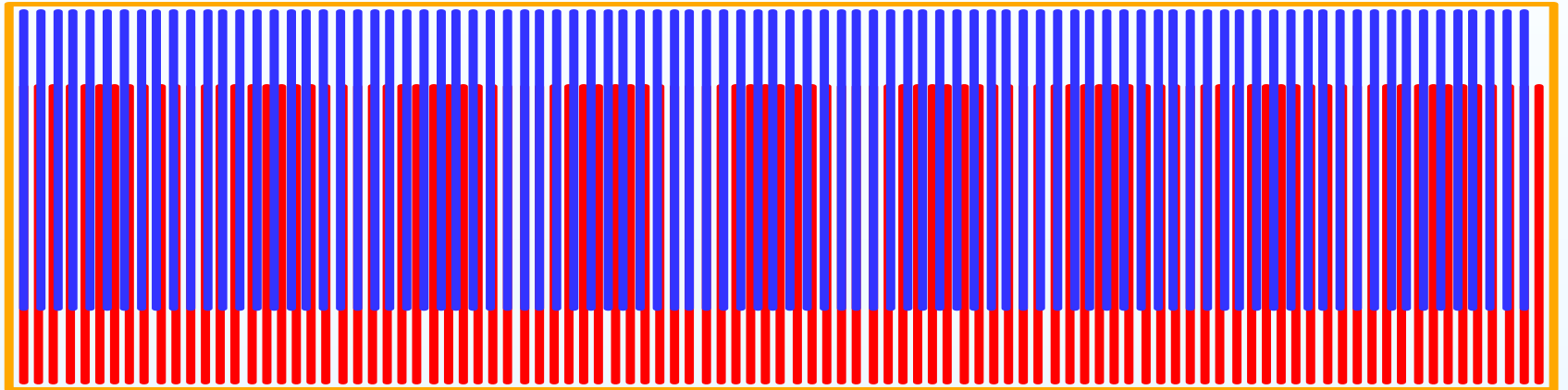


$$\omega_1$$

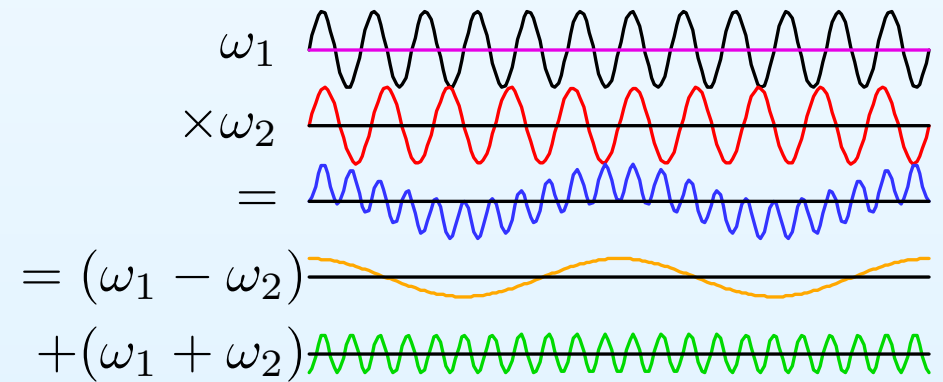
$$\implies \omega_3 = 0.1\omega_1 = \omega_2 - \omega_1$$

# Multiplicación de Ondas

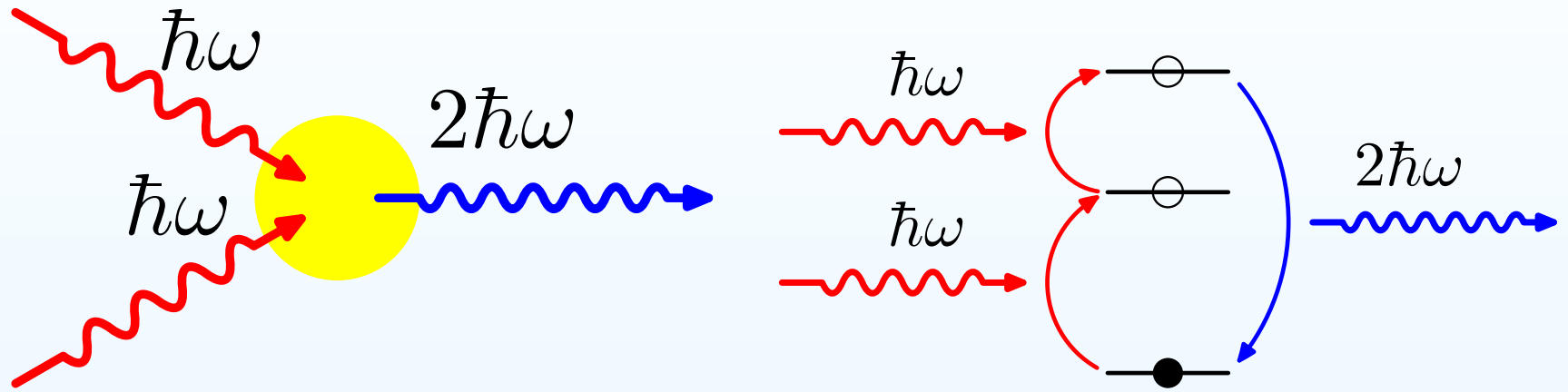
$$\omega_2 = 1.1\omega_1$$



$$\omega_1$$
$$\implies \omega_3 = 0.1\omega_1 = \omega_2 - \omega_1$$



# Generación de Segundo Armónico



$$\vec{P}(\vec{2}\omega) \propto \vec{E}(\omega)\vec{E}(\omega)$$

# GSA y Simetría

$$\vec{P}^{(2)} = \chi^{(2)} \vec{E} \vec{E}$$

## GSA y Simetría

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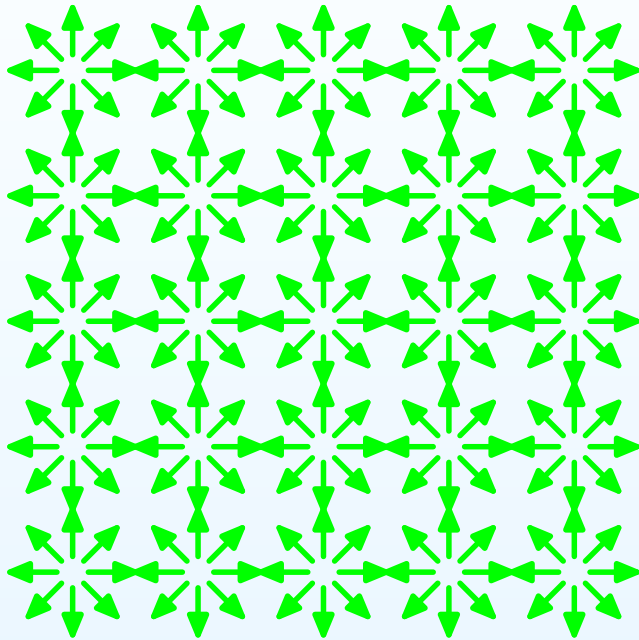
$$\vec{P}^{(2)} = \chi^{(2)} \vec{E} \vec{E}$$

Después de una inversión

$$-\vec{P}^{(2)} = \chi^{(2)} (-\vec{E})(-\vec{E})$$



## GSA y Simetría



$$\vec{P}^{(2)} = \chi^{(2)} \vec{E} \vec{E}$$

Después de una inversión

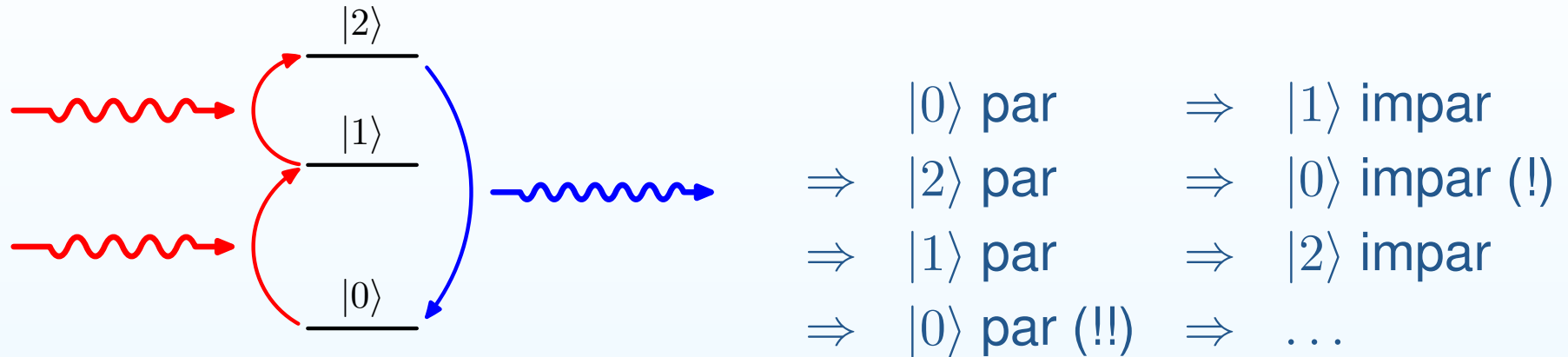
$$-\vec{P}^{(2)} = \chi^{(2)} (-\vec{E})(-\vec{E})$$

Centrosimetría  $\Rightarrow$

$$\chi^{(2)}(\text{antes}) = \chi^{(2)}(\text{después})$$

$$\Rightarrow \vec{P}^{(2)} = 0, \quad \chi^{(2)} = 0$$

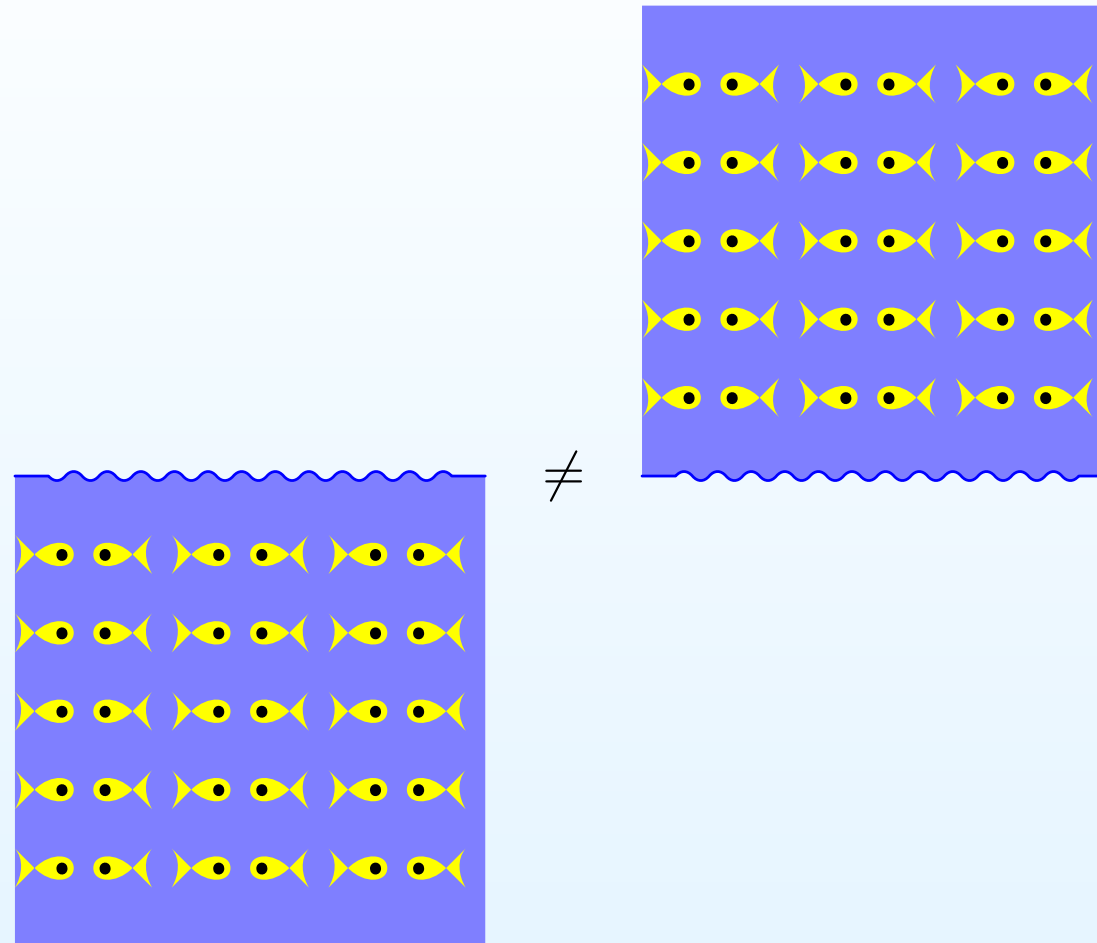
# Centrosimetría y Paridad



$$\chi^{(2)} \propto \langle 0|\hat{p}|2\rangle \langle 2|\hat{p}|1\rangle \langle 1|\hat{p}|0\rangle = 0$$

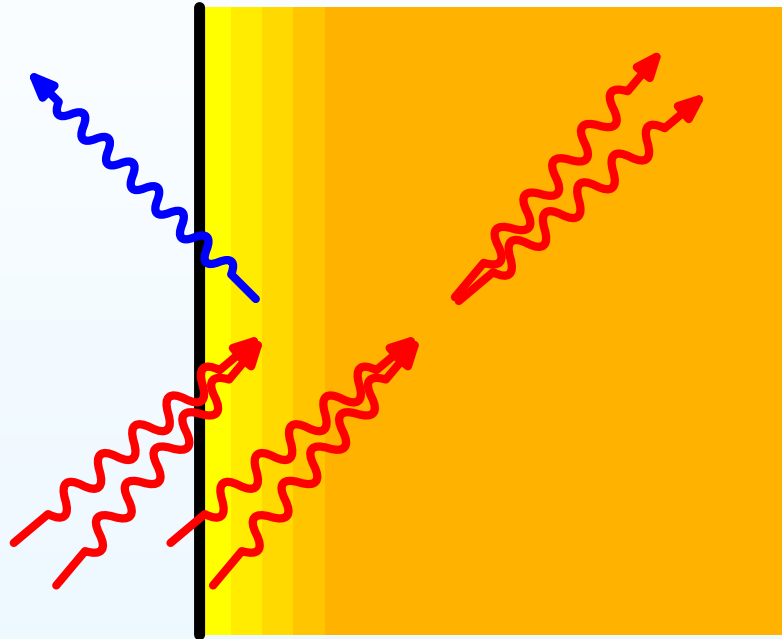
La susceptibilidad *dipolar* de segundo orden es nula.

# Centrosimetría y Superficies



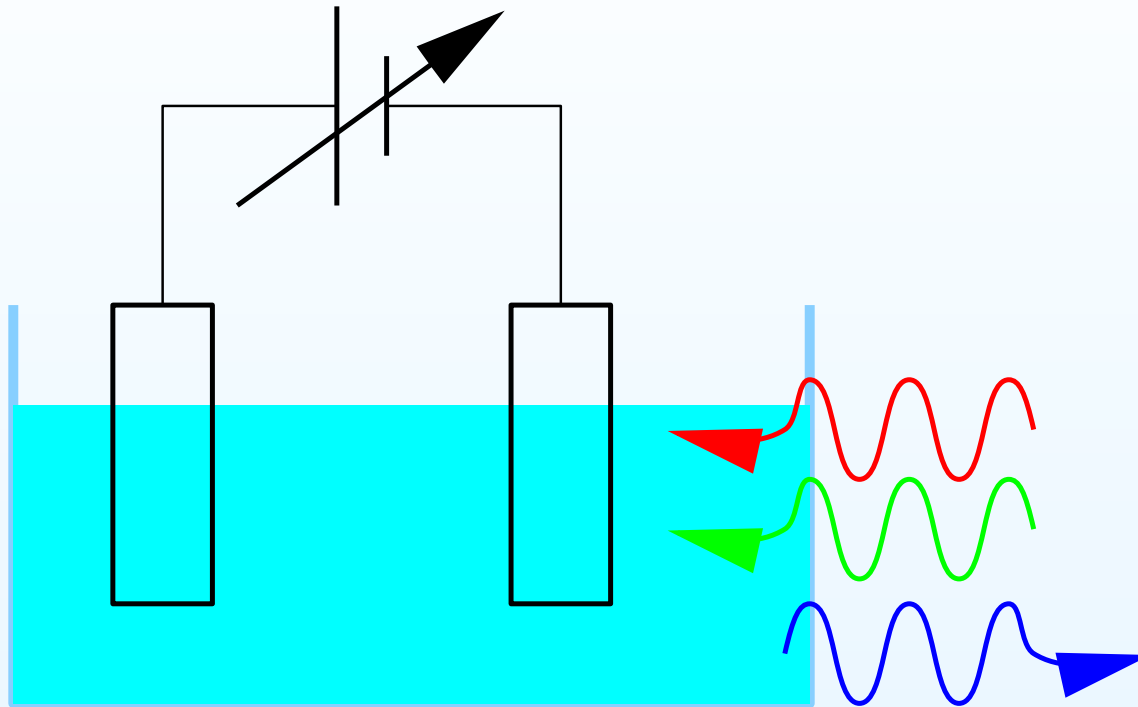
¡Las superficies no son centrosimétricas!

# GSA y Superficies

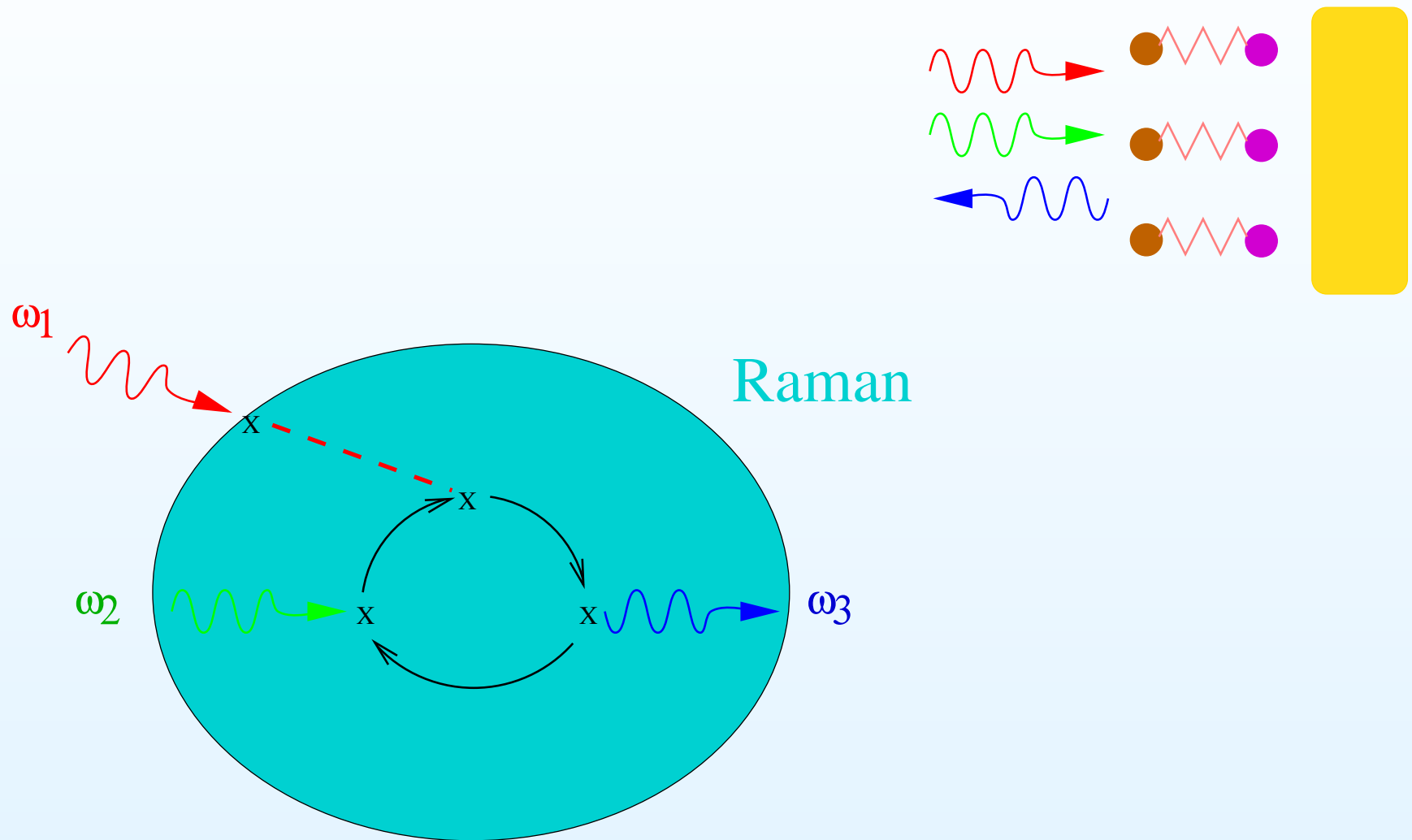


La GSA dipolar  $P_i^{(2)} = \chi_{ijk} E_j E_k$  viene de las superficies.  
Puede haber GSA multipolar en el bulto  $P_i^{(2)} = \chi_{ijkl} E_j \partial_k E_l$ .

# Espectroscopías ópticas de superficies



# Adsorbatos

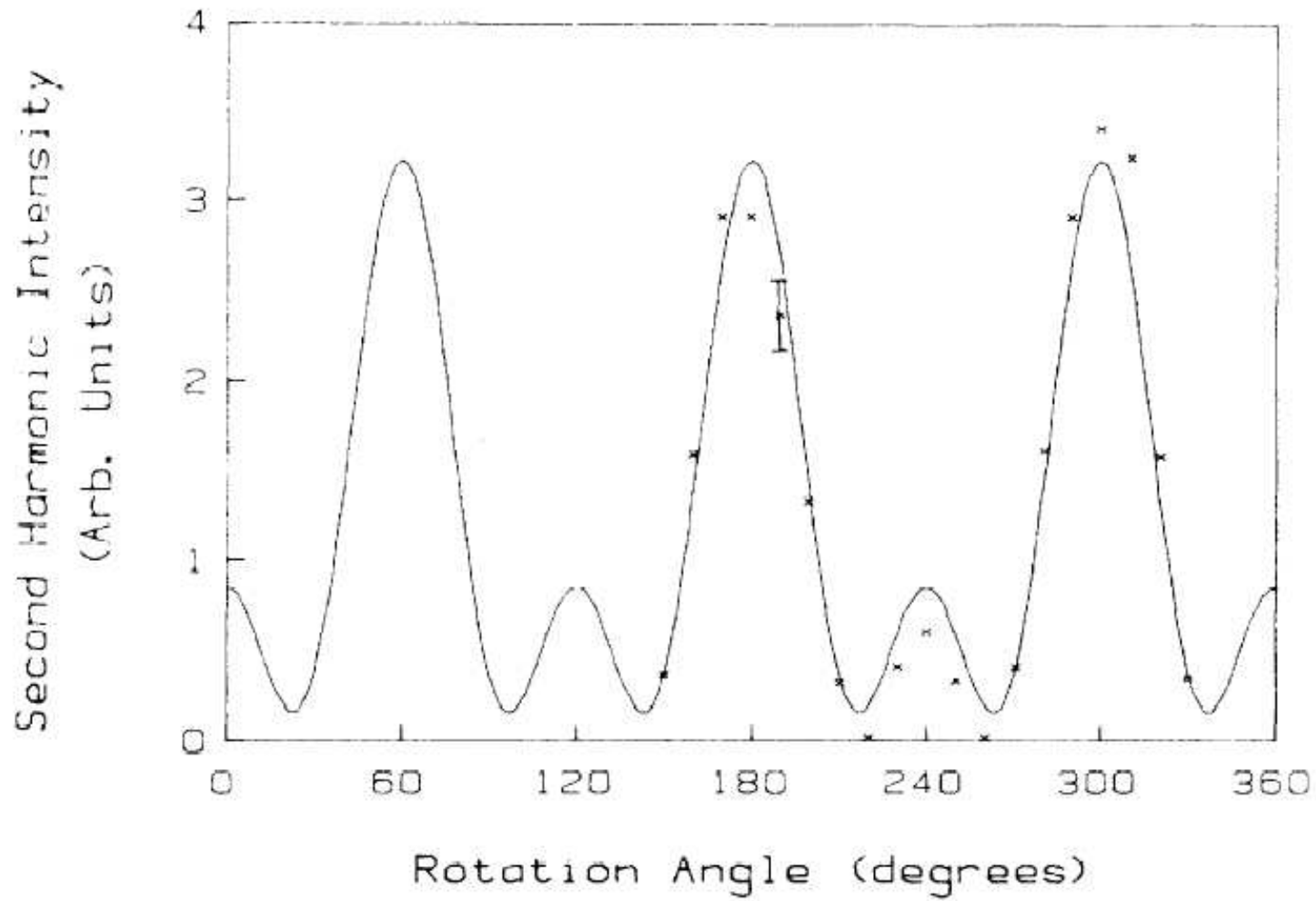


# Simetría

$\chi_{ijk} = S_{ii'} S_{jj'} S_{kk'} \chi_{i'j'k'} \Rightarrow$  sólo algunas componentes de  $\chi_{ijk}$  pueden ser no-nulas. Con la superficie normal a  $z$ ,

Simetría	$\chi_{ijk}$ no-nulas
1	$xxx, xxy, xyx, yxx, yxy, yyx, xxz, xyz, yxz, yyz, zxx, zxy, zyy, xzz, yzz, zxz, zyz, zzz$
1m ( $\perp y$ )	$xxx, xyy, xzz, xzx, yzy, yxy, zxx, zyy, zxz, zzz$
2	$xzx, xyz, yxz, yzy, zxx, zyy, zxy, zzz$
2mm	$xzx, yzy, zxx, zyy, zzz$
3	$xxx = -xyx = -yxx, yyy = -yxx = -xyx, yzy = xzx, zxx = zyy$
3m ( $\perp y$ )	$xxx = -xyx = -yxx, xzx = yzy, zxx = zyy, zzz$
4, 6, $\infty$	$xxz = yyz, zxx = zyy, xyz = -yxz, zzz$
4mm, 6mm, $\infty m$	$xxz = yyz, zxx = zyy, zzz$

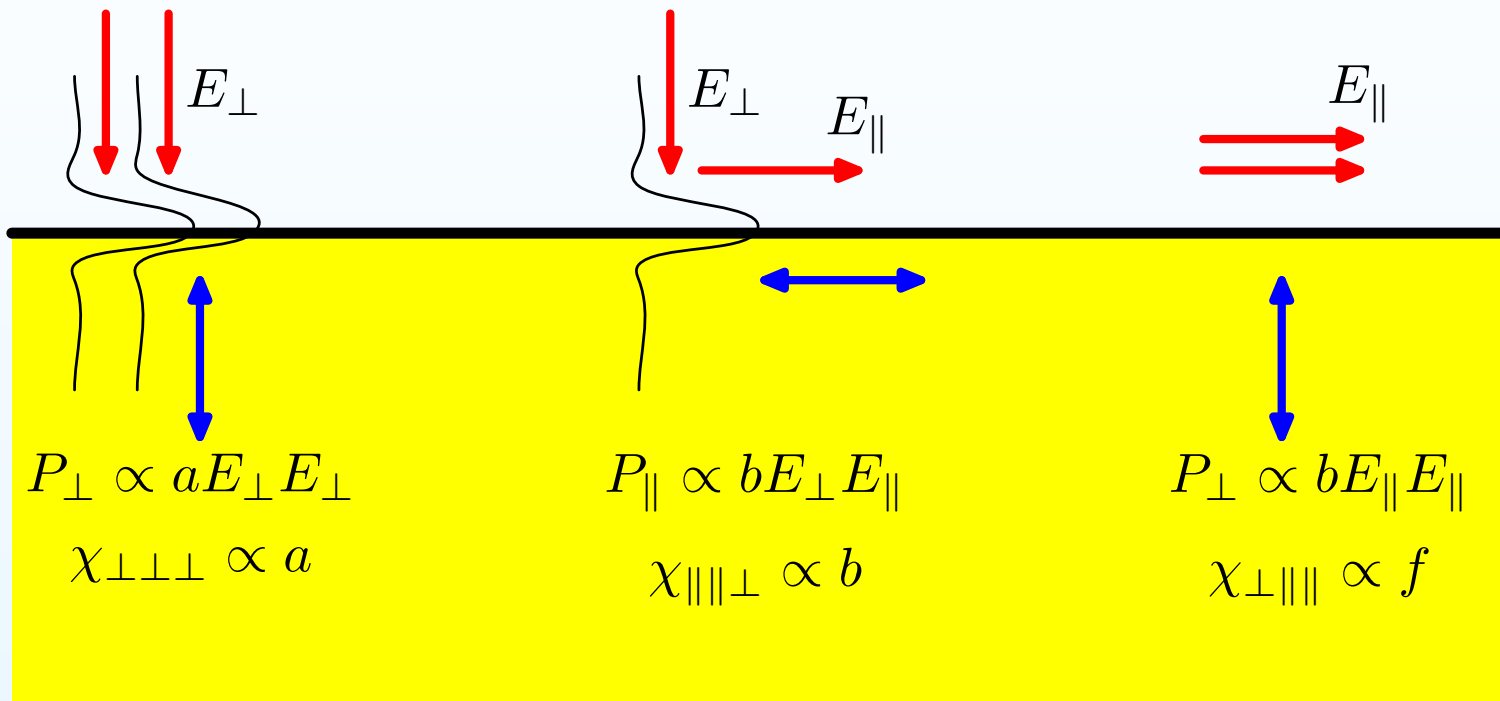
# Ejemplo: Si(111) $2 \times 1, s \rightarrow p$



(Sipe et al., 1987)



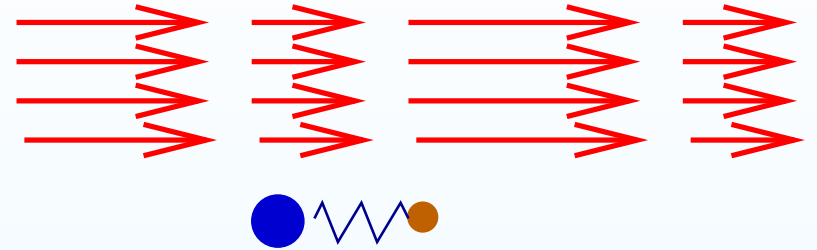
# Respuesta no lineal de la superficie



Superficie:  $P_i = \chi_{ijk} E_j E_k$

Bulto:  $P_i \propto \vec{E} \cdot \nabla E, \vec{E} \times \vec{B}, \nabla E^2$

## Cálculo de $\chi_{ijk}$



Oscilador armónico

$$\vec{E}(\vec{r}) = \vec{E}(0) + \vec{r} \cdot \nabla \vec{E}(0) + \dots$$

$$m\ddot{\vec{r}} = -e\vec{E}(0, t) - m\omega_0^2\vec{r} - \frac{m}{\tau}\dot{\vec{r}} \\ -e\vec{r} \cdot \nabla \vec{E}(0, t) - \frac{e}{c}\dot{\vec{r}} \times \vec{B}(\vec{0}, t)$$

$\Rightarrow$  oscilador paramétrico si  $\vec{E} \neq$  homogéneo.

## Respuesta de una molécula

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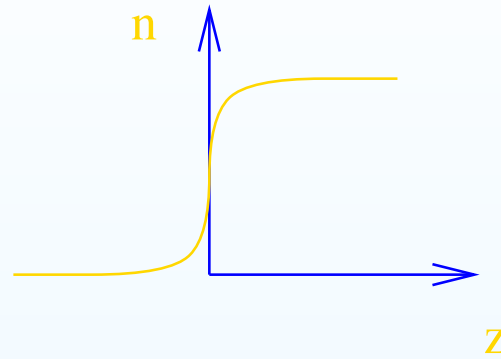
$$\vec{p}^{(1)} = \alpha(\omega) \vec{E}(0, 1)$$

$$\alpha(\omega) = \frac{e^2/m}{\omega_0^2 - \omega^2 - i\omega/\tau}$$

$$\vec{p}^{(2)} = -\frac{1}{2e} \alpha(\omega) \alpha(2\omega) [\nabla E^2 - 4\vec{E} \times (\nabla \times \vec{E})]$$

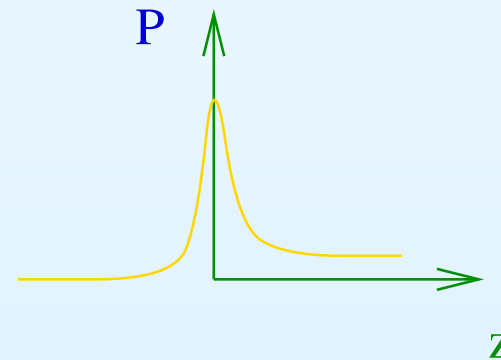
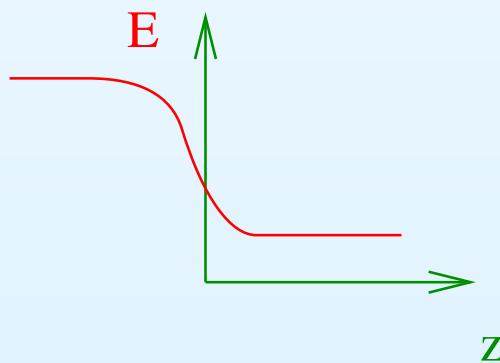
$$\vec{Q}^{(2)} = -\frac{1}{e} \alpha(\omega)^2 \vec{E}_i \vec{E}_j$$

# Respuesta de la superficie



$$\vec{P}^{(2)} = n\vec{p}^{(2)} - \frac{1}{2}\nabla \cdot n \vec{Q}^{(2)}$$

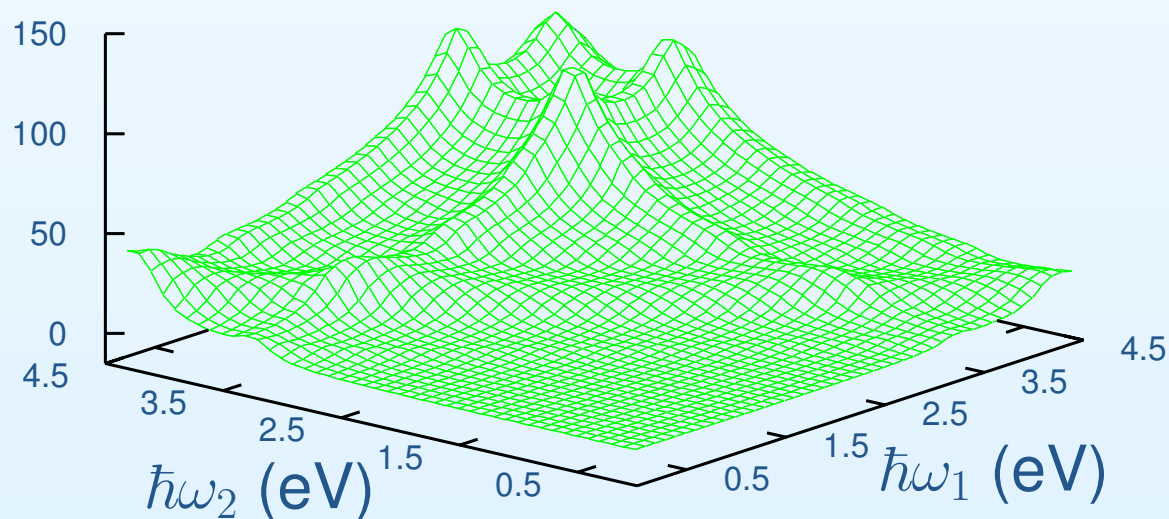
$$\vec{P}_s^{(2)} = \int dz \vec{P}^{(2)}$$



# $\chi_{zzz}$

$$\chi_{zzz}^s(\omega) = \frac{\alpha^2(\omega) \alpha(2\omega) \log(\epsilon^B(\omega)/\epsilon^B(2\omega))}{8\pi e (\alpha(\omega) - \alpha(2\omega))^2} + \frac{\alpha(\omega) \epsilon^B(\omega) - 1}{8\pi e \epsilon^B(\omega)} \left( \frac{1}{\epsilon^B(\omega)} + \frac{\alpha(2\omega)}{\alpha(2\omega) - \alpha(\omega)} \right),$$

$10^{20} \mathcal{R}_{ppp}$  (cm<sup>2</sup>/W)



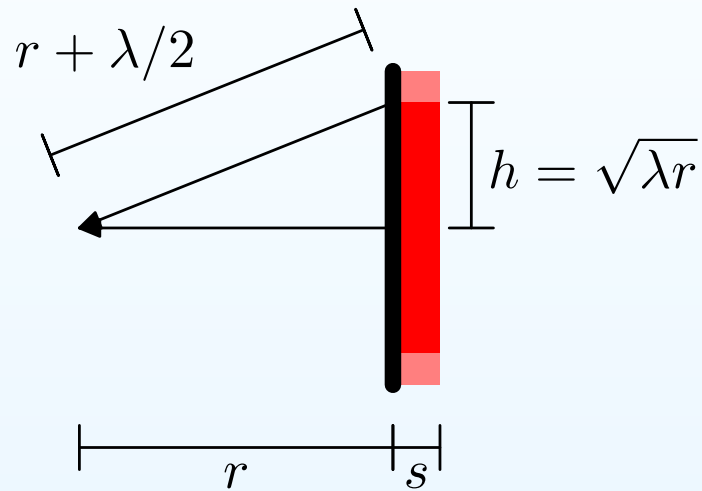
Silicio

## Eficiencia

- $E = \frac{p}{r^3} \rightarrow \frac{p}{\lambda^2 r}$

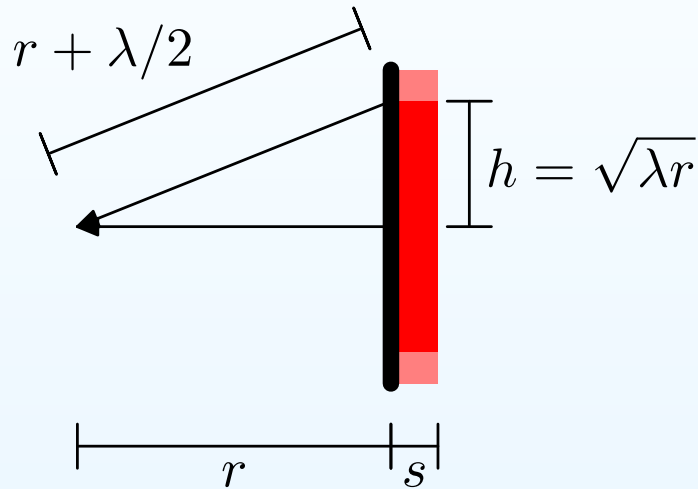
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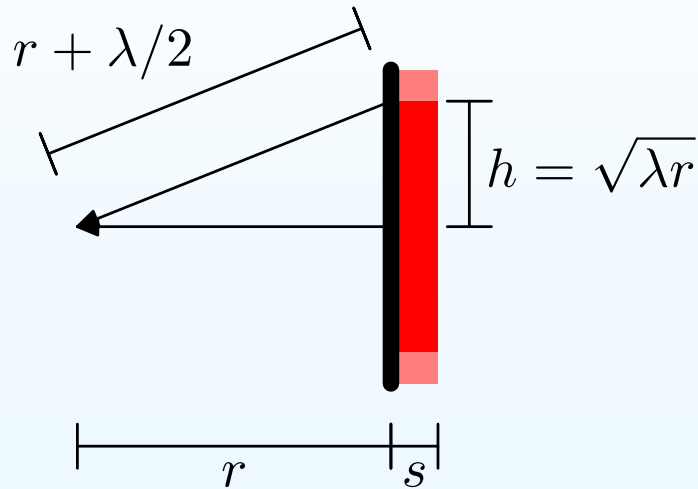


- $p \approx Ph^2 s \approx \frac{E^2}{e/a_B^2} \lambda r a_B$



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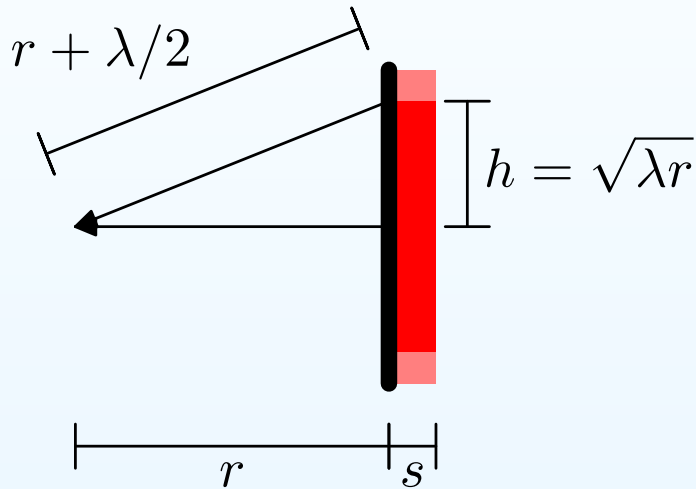


- $p \approx Ph^2 s \approx \frac{E^2}{e/a_B^2} \lambda r a_B$

- $E \approx \frac{a_B^3}{\lambda e} E^2$

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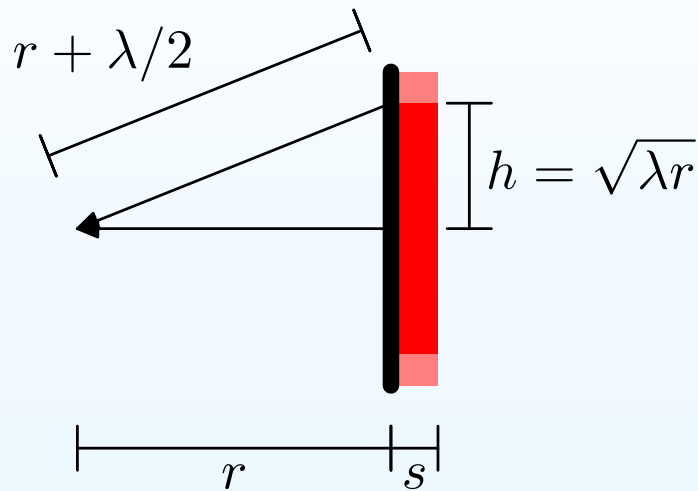
- $p \approx Ph^2 s \approx \frac{E^2}{e/a_B^2} \lambda r a_B$

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- $I \approx cE^2 = RI^2$

# Eficiencia

- $E = \frac{p}{r^3} \rightarrow \frac{p}{\lambda^2 r}$



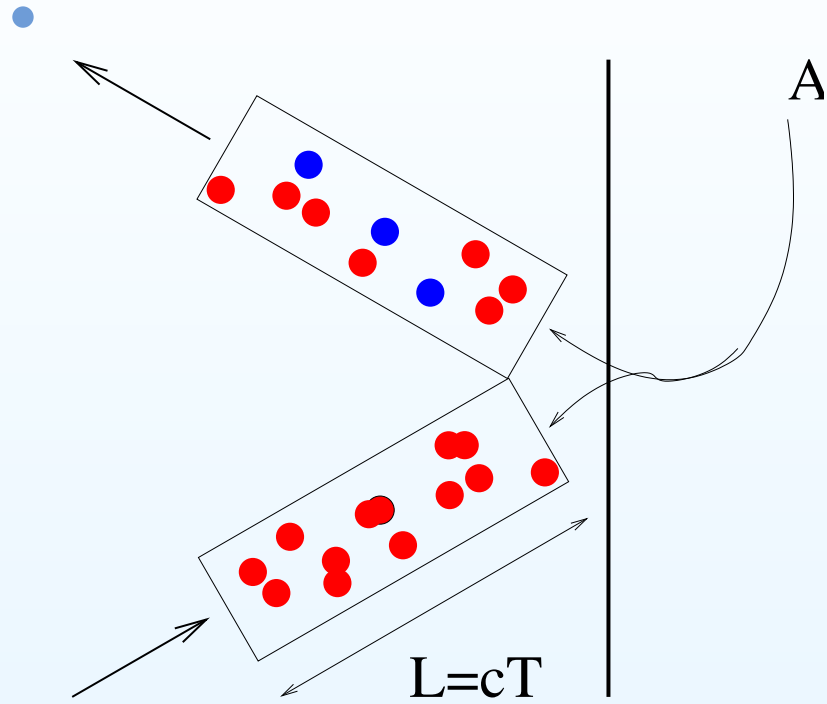
- $p \approx Ph^2 s \approx \frac{E^2}{e/a_B^2} \lambda r a_B$

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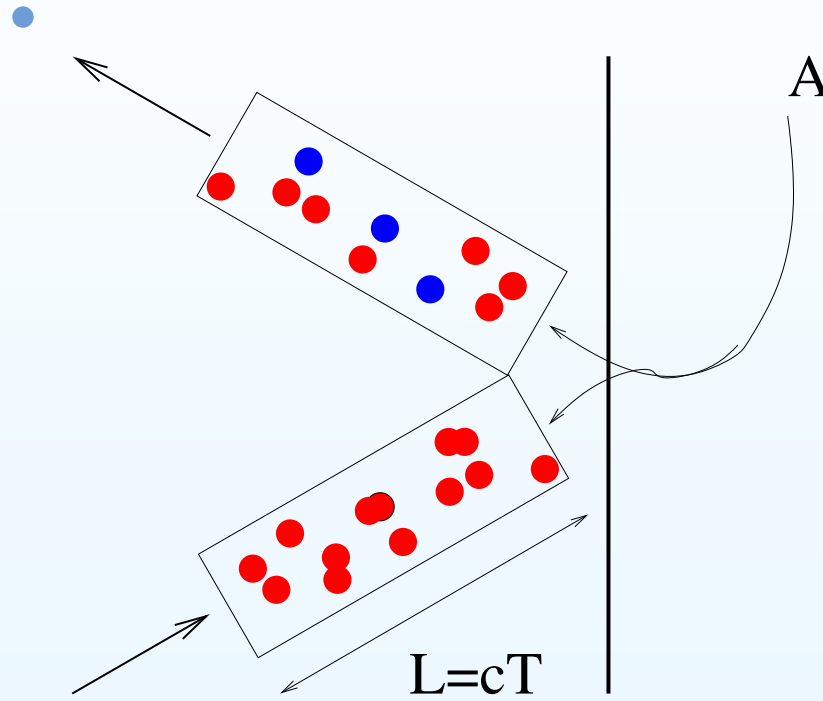
- $I \approx cE^2 = RI^2$

- $R \approx \left(\frac{a_B}{\lambda}\right)^2 \frac{a_B}{e^2} \frac{a_B^3}{c} \approx 10^{-23} \text{cm}^2/\text{W}$

# Tamaño de un fotón

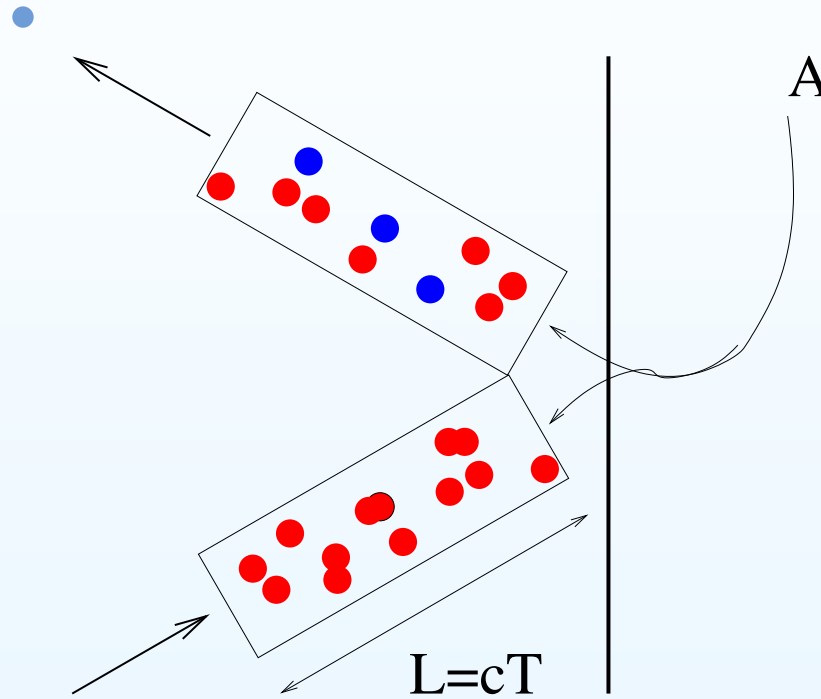


# Tamaño de un fotón



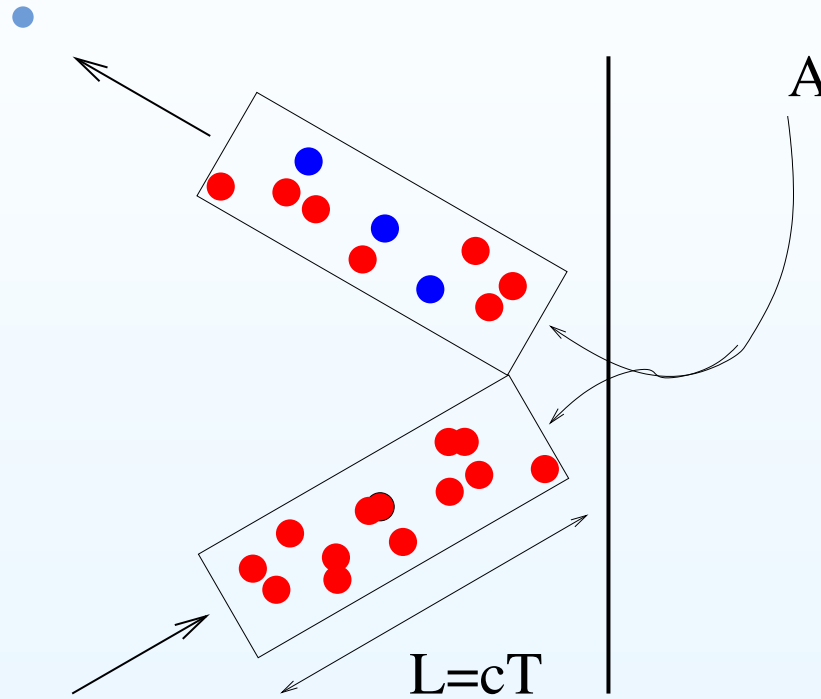
- $I \approx N\hbar\omega/AT$

# Tamaño de un fotón



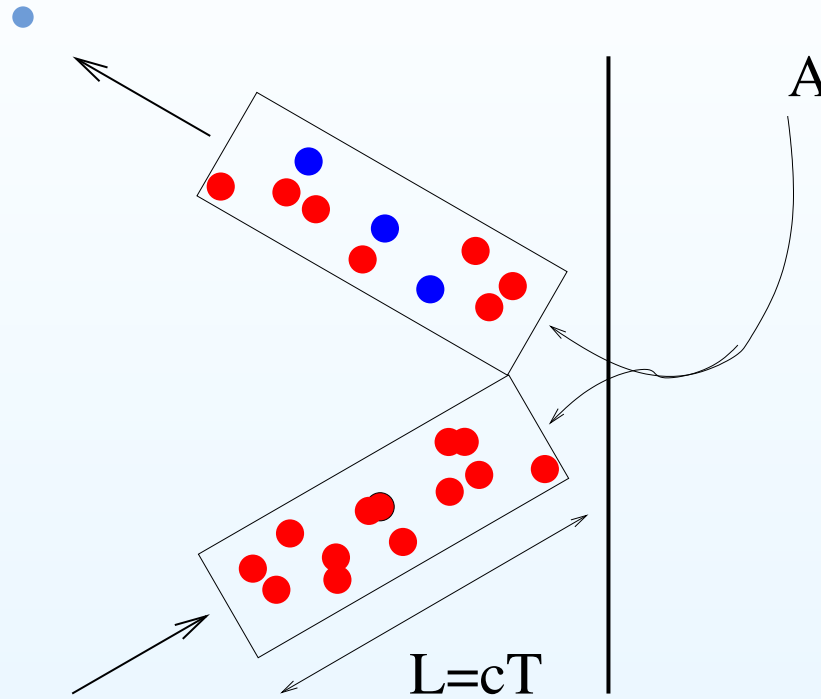
- $I \approx N\hbar\omega/AT$
- $I \approx 2N\hbar\omega/AT$

# Tamaño de un fotón



- $I \approx N\hbar\omega/AT$
- $I \approx 2N\hbar\omega/AT$
- $N = \frac{\Omega}{V} N^2$

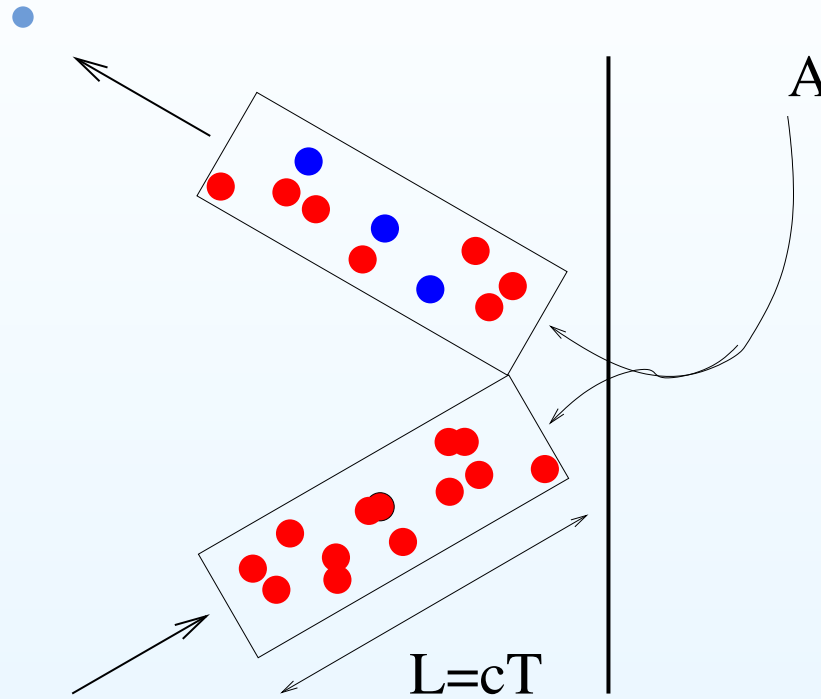
# Tamaño de un fotón



- $I \approx N\hbar\omega/AT$
- $I \approx 2N\hbar\omega/AT$
- $N = \frac{\Omega}{V} N^2$
- $V = AcT$

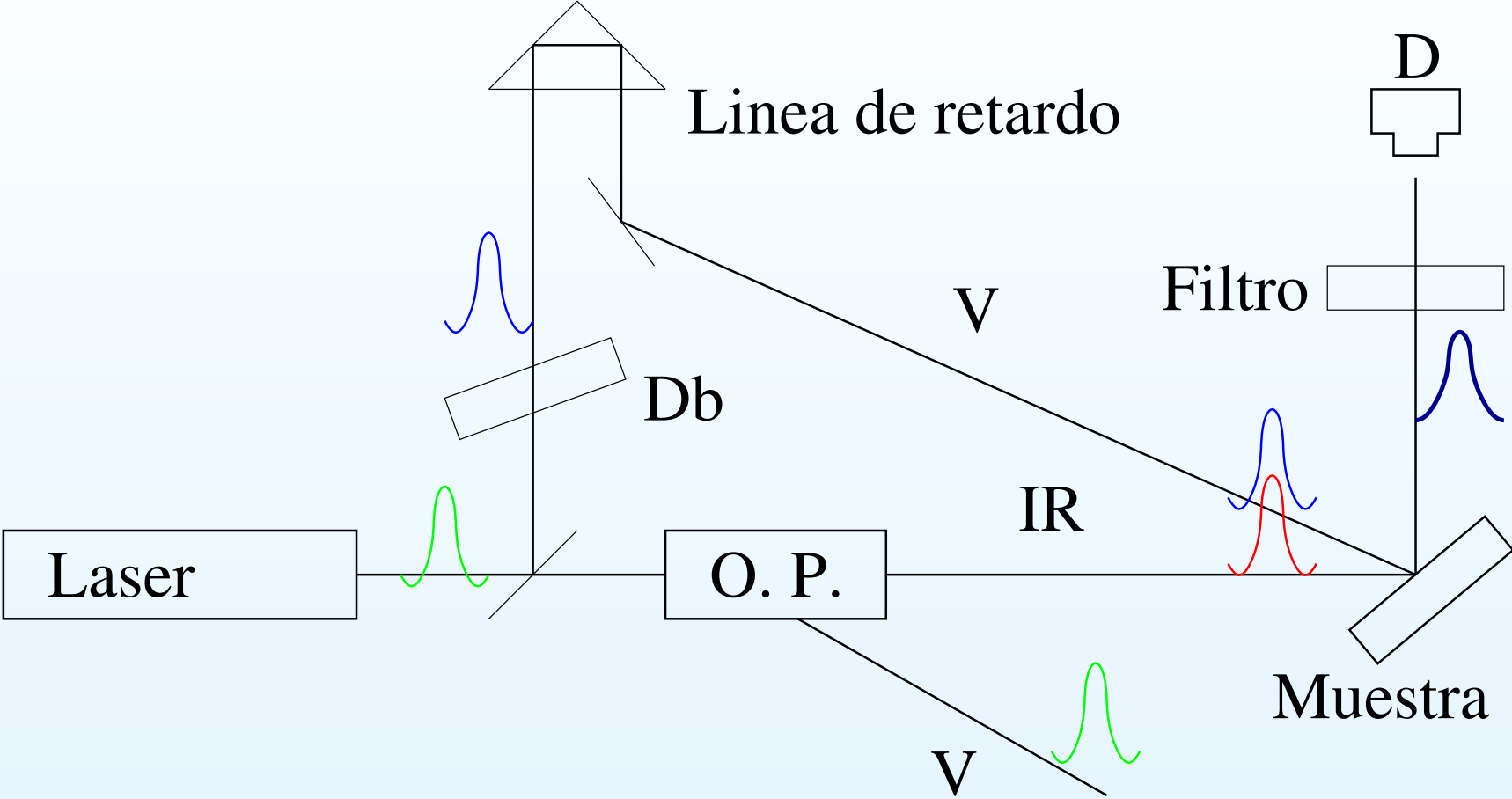


# Tamaño de un fotón



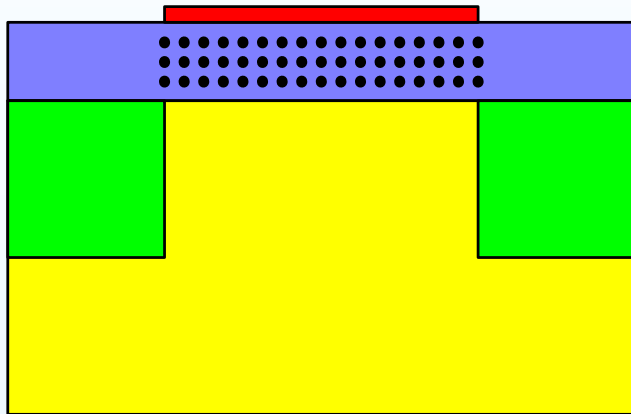
- $I \approx N\hbar\omega/AT$
- $I \approx 2N\hbar\omega/AT$
- $N = \frac{\Omega}{V} N^2$
- $V = AcT$
- $\Omega = \frac{\hbar c}{e^2} \left(\frac{a_B}{\lambda}\right)^3 a_B^3 \approx 10^{-7} a_B^3$

# Diagrama SFG

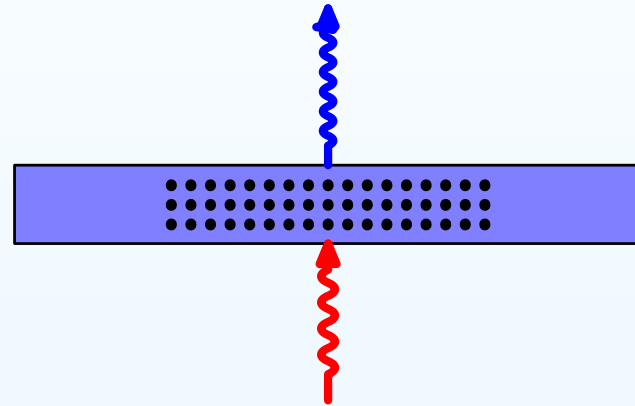


# GSA por Nanopartículas

Memorias flash



Observa superficie con GSA



# Experimento

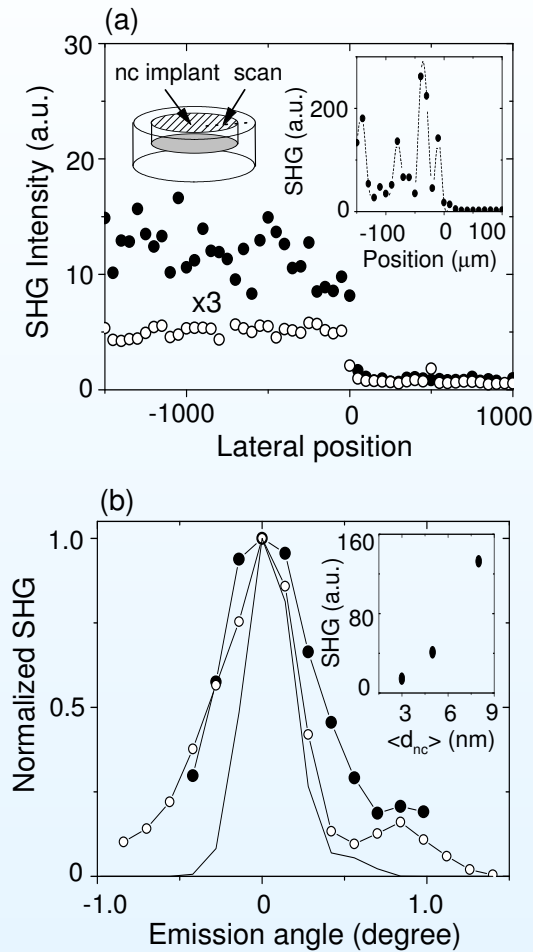
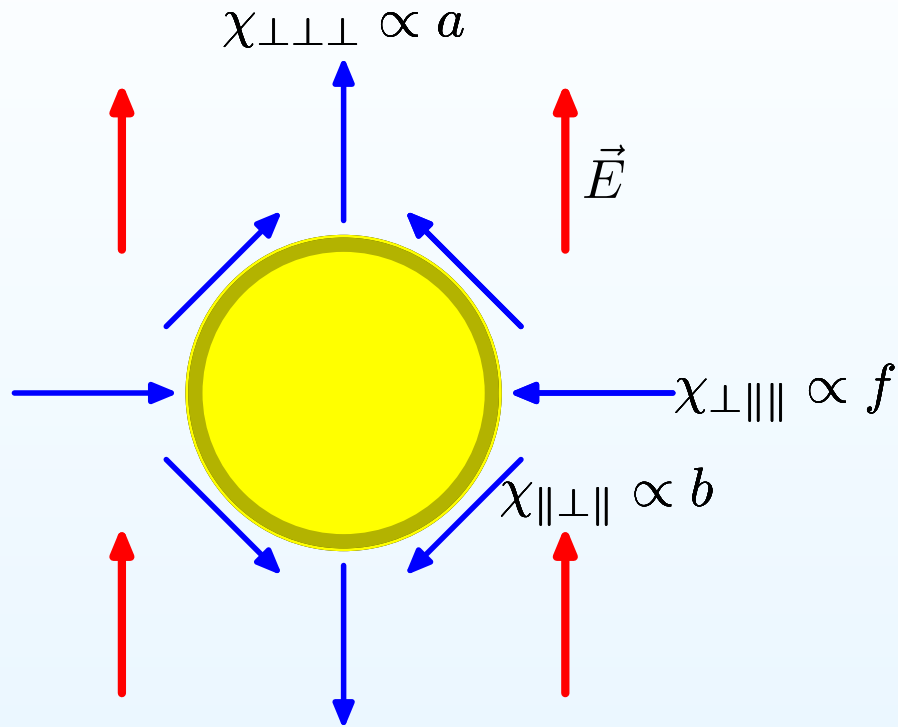


FIG. 3

Y. Jiang, P. T. Wilson, M. C. Downer, C. W. White, and S. P. Withrow, *Appl. Phys. Lett.* **78**, 766 (2001).

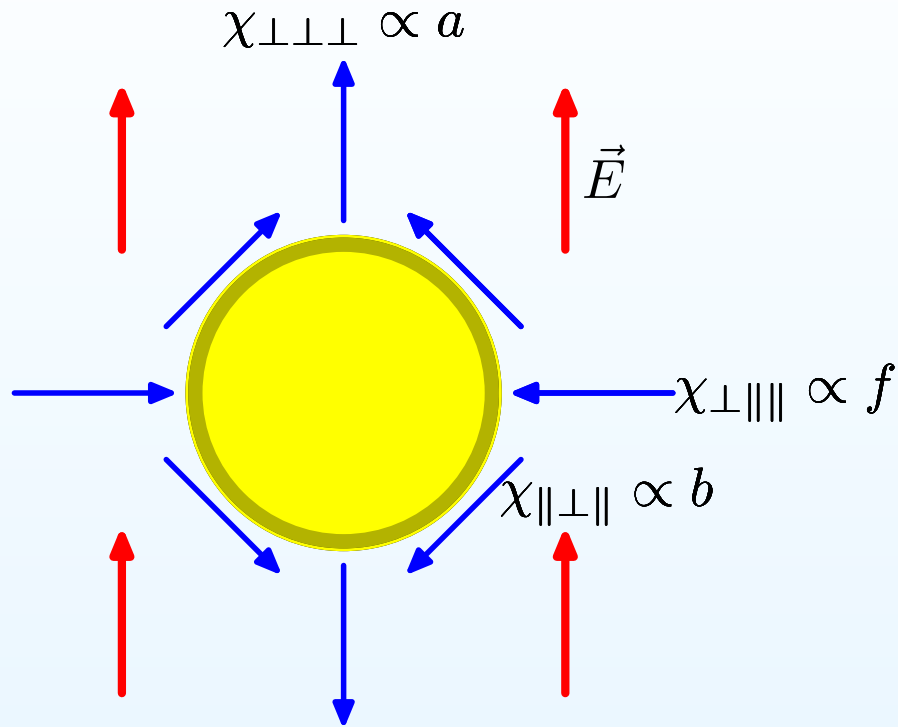
- La señal viene de las nanoesferas.
- Es sensible a la interface (recocido en Ar vs. Ar/H<sub>2</sub>).
- GSA hacia el frente.
- Orilla vs. bulto.

# GSA de una esfera



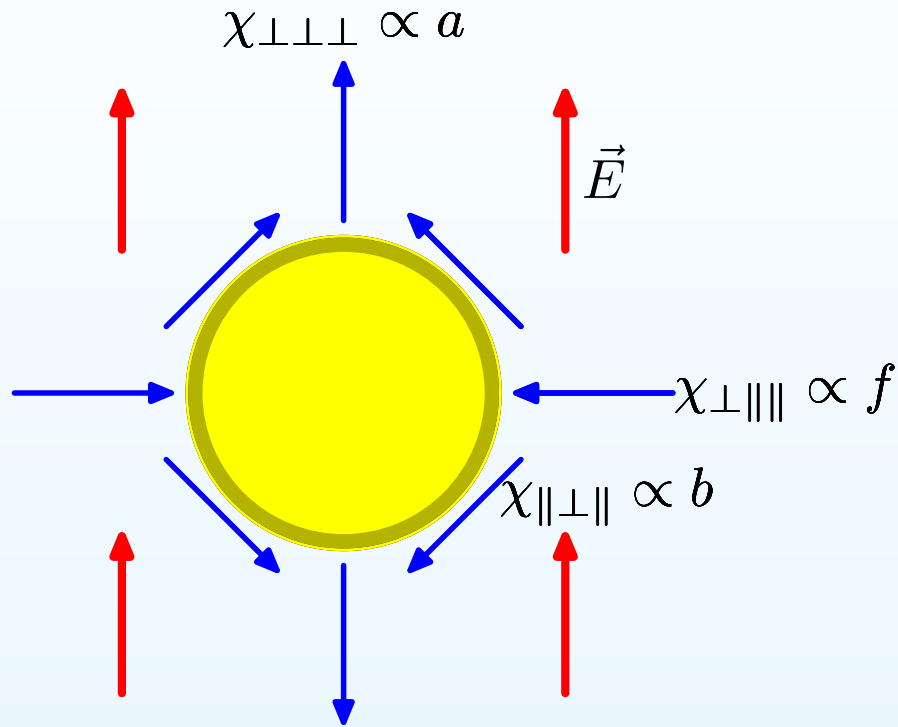
- La centrosimetría se pierde localmente...

# GSA de una esfera



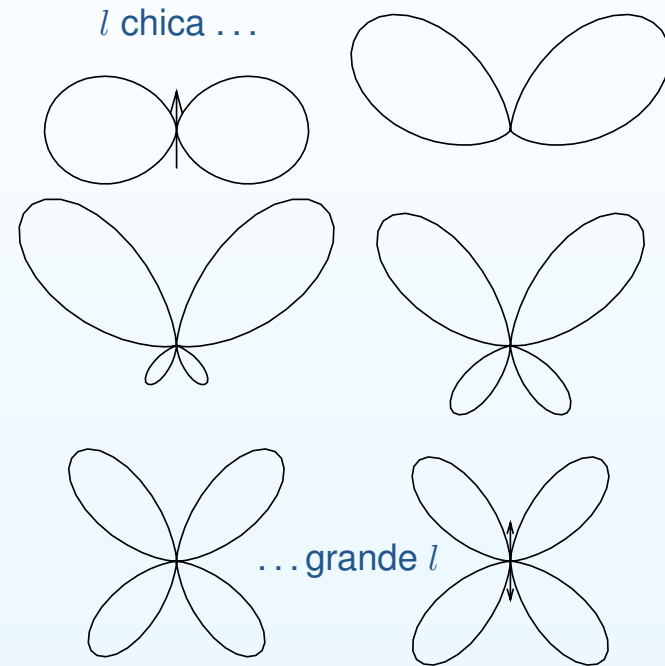
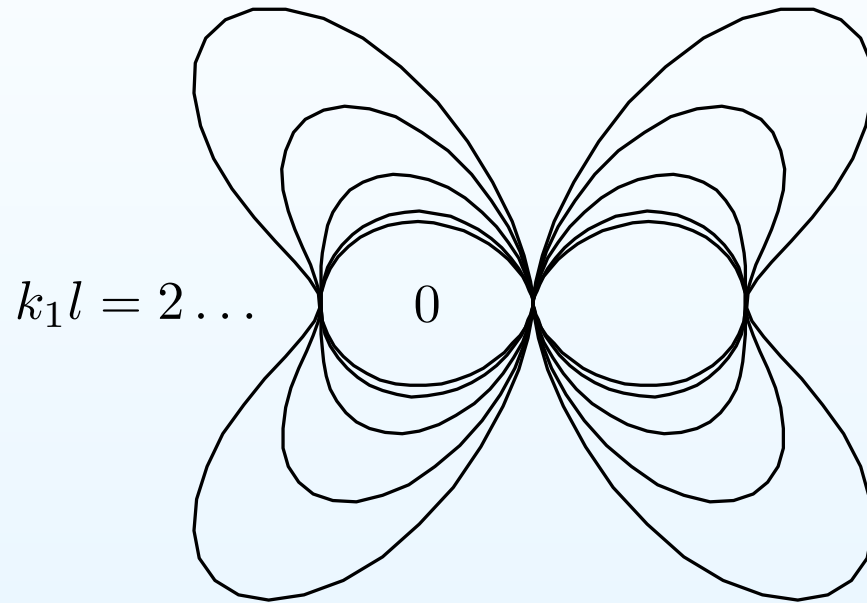
- La centrosimetría se pierde localmente...
- pero se recupera globalmente.

# GSA de una esfera



- La centrosimetría se pierde localmente...
- pero se recupera globalmente.
- El dipolo total es nulo.

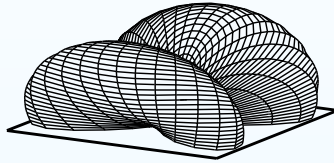
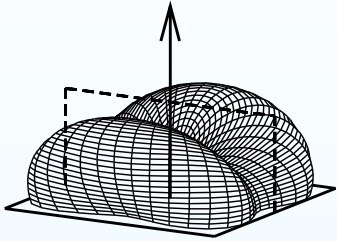
# Radiación dipolar vs. cuadrupolar



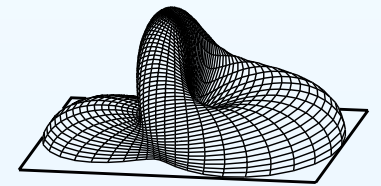
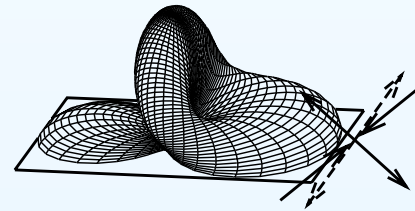
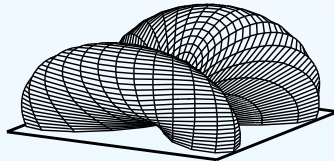
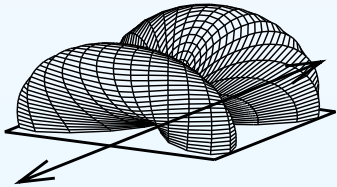
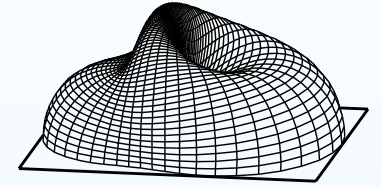
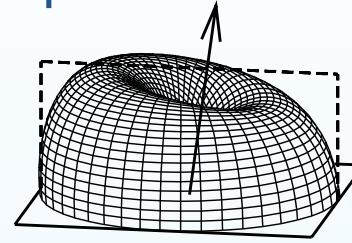


# Eficiencia de GSA para nanoesfera sobre sustrato

polarización  $s \rightarrow p$



polarización  $p \rightarrow p$



# Comparación

Cero radiación frontal y  
Distribución ancha  
vs.  
¡Distribución angosta  
alrededor de la dirección  
frontal!

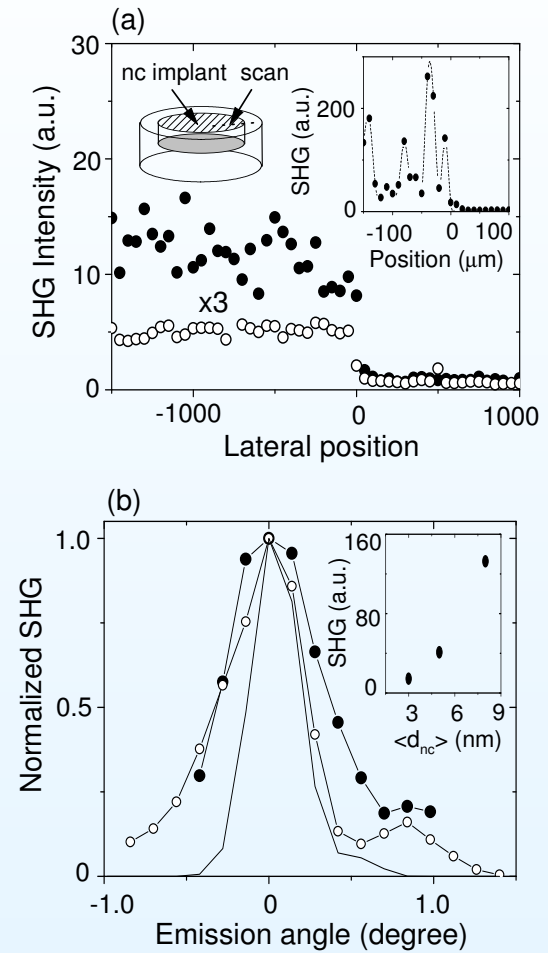
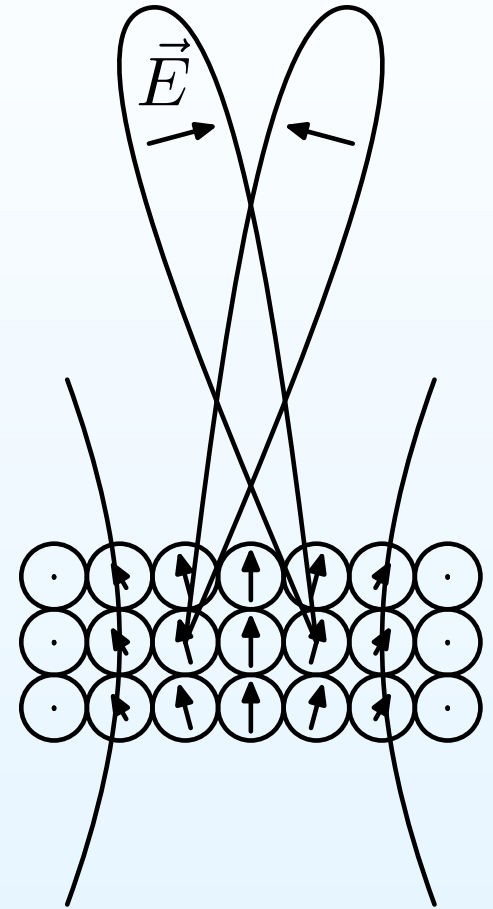
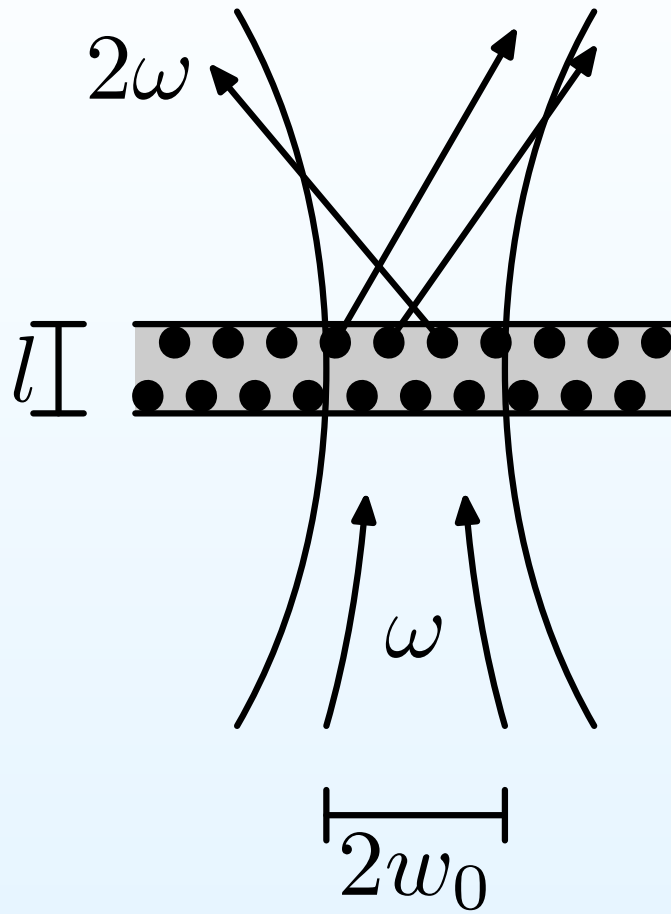


FIG. 3

# GSA de una película compuesta

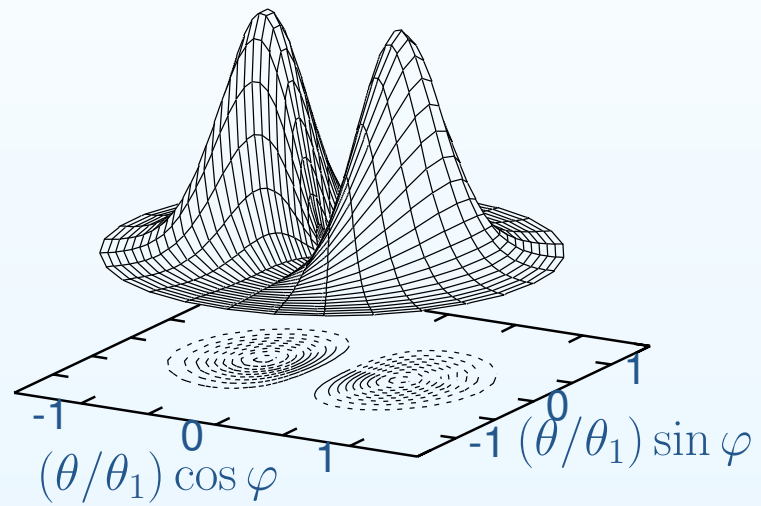


# Teoría

$$\begin{aligned}\vec{P}^{nl} &= n_s \vec{p}^{(2)} - \frac{1}{6} \nabla \cdot \vec{Q}^{(2)} && \implies \vec{j}^{(2)} \\ &= \Gamma \nabla E^2 + \Delta' \vec{E} \cdot \nabla \vec{E} && \implies \vec{A}^{(2)} \\ & && \implies \vec{E}^{(2)}, \vec{B}^{(2)} \\ \Gamma &= \frac{n_b}{18} (9\gamma^m + \gamma^q - 3\tilde{\gamma}^q) && \implies \vec{S}^{(2)} \\ \Delta' &\equiv n_b (\gamma^e - \gamma^m - \gamma^q/6), && \implies \frac{d\mathcal{E}}{d\Omega} = \frac{1}{\mathcal{P}^2} \frac{dI^{(2)}}{d\Omega}\end{aligned}$$

# Distribución angular

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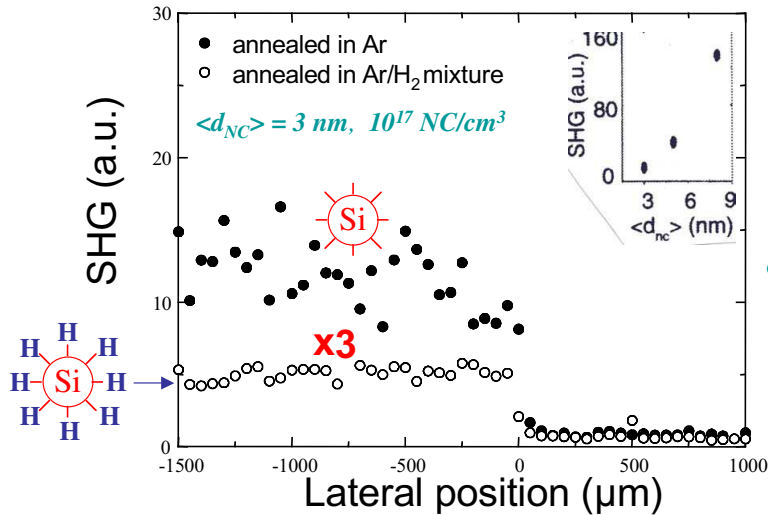
# Experimento

## Single wavelength SHG scan across boundary between nc-Si implanted glass & unimplanted glass

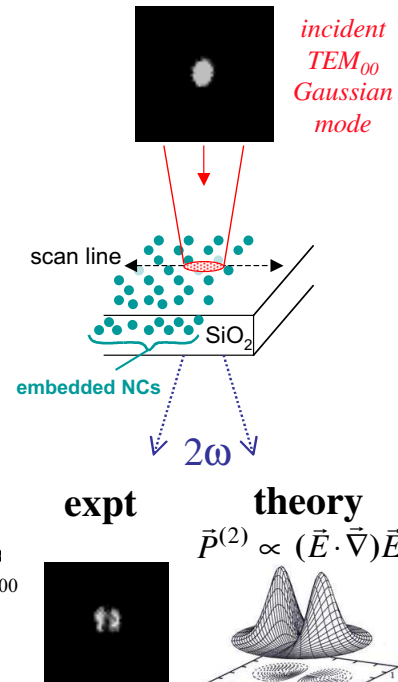
presence & size of Si NCs

SHG sensitive to: Si/SiO<sub>2</sub> interface chemistry

local particle density gradients



Jiang *et al.*, APL 78, 766 (2001)



Jiang (03)

Brudny, PRB 62, 11152 (00)  
Mochan, (03).

## Eficiencia

$$\begin{aligned}\mathcal{E} &= 10^{-2} \zeta (qa_B)^4 (ql)^2 f_b^2 \theta_1^4 \frac{1}{e^2/a_B} \frac{1}{c/a_B} \\ &\approx 10^{-4} \zeta (qa_B)^4 (ql)^2 f_b^2 \theta_1^4 \mathbf{W}^{-1} \\ &\approx 10^{-24} \mathbf{W}^{-1}.\end{aligned}$$

Como la polarización inducida es proporcional a  $\vec{E} \nabla \vec{E} \sim E^2 / w_0$ , la eficiencia es proporcional a la **intensidad** que arriba, ¡no a la potencia!

¡Más poder no es necesariamente una mejora!

## Conclusiones

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- El mezclado de tres ondas conduce a espectroscopías óptica sensible a superficies.
- La señal del bulto es fuertemente suprimida por centrosimetría.
- La eficiencia es muy pequeña.
- La simetría de la superficie puede observarse directamente.
- El modelo de *dipolium* conduce a expresiones analíticas y es una buena primera aproximación.
- Se puede observar la superficie de nanoesferas aisladas, depositadas en superficies y en medios compuestos.



## Conclusiones

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- Las contribuciones cuadrupolares y dipolares pueden ser comparables, dando origen a patrones complejos de radiación.
- No hay radiación frontal, pero si cercana.
- La eficiencia en medios compuestos no puede incrementarse simplemente aumentando la potencia incidente.